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Beliefs and Mathematical School Knowledge at the Beginning of Pre-service Primary Teacher Education

Creencias y conocimiento matemático escolar al comienzo de la formación inicial docente en estudiantes de Pedagogía General Básica

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Abstract

The Teacher Professional Development System, existing in Chile since 2016, requires universities with teacher training programs to make a diagnosis in the first year of the degree. In this context, we present a study that allowed us to describe the beliefs regarding mathematics, it's teaching and learning, as well as the mathematical school knowledge corresponding to the levels from 1 to 6, which have the future primary education teachers, considering that it is the mathematics that they should teach in their professional life. This diagnosis was made by applying a pencil and paper questionnaire to a stratified random sample of 511 first-year primary school students from 14 universities in Chile. From this, we develop a characterisation of students regarding their beliefs and knowledge of school mathematics, investigating the relationships between them. The results show that students consider mathematical activity differently, and have greater mastery of arithmetic content.

Keywords: beliefs, mathematics education, mathematical knowledge, pre-service teacher education.

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Resumen

El Sistema de Desarrollo Profesional Docente existente en Chile desde 2016, obliga a las universidades con programas de formación de profesores a realizar un diagnóstico en el primer año de la carrera. En este contexto, el estudio describe las creencias que tienen los futuros profesores de educación primaria respecto de la matemática, su enseñanza y aprendizaje, así como el conocimiento matemático escolar correspondiente a los niveles de 1º a 6º año básico (6 a 12 años), considerando que es la materia que deberán enseñar en su vida profesional. Este diagnóstico se realizó mediante la aplicación de un cuestionario de lápiz y papel, a una muestra aleatoria estratificada de 511 estudiantes de primer año de educación primaria de 14 universidades chilenas. A partir de esto, desarrollamos una caracterización de los estudiantes en términos de sus creencias y conocimiento de matemática escolar, indagándose en las relaciones entre estos dos aspectos. Los resultados muestran que los estudiantes consideran la actividad matemática de forma diversa, y que evidencian un dominio mayor de los contenidos de aritmética por sobre los otros conocimientos matemáticos..

Palabras clave: conocimiento matemático, creencias, educación matemática, formación inicial docente.

Presentation

During the last decade, Chile has developed a number of programs aimed at strengthening pre-service teacher education (PTE) along with strategies for supporting new teachers' entry to the profession. Nowadays, the Teacher Professional Development System, created through Law 20.903, includes several initiatives, including an obligatory test at the beginning of teachers' professional career. Test results influence the accreditation of Teacher Education degrees offered by Chilean universities, but have no direct consequences for pre-service teachers. In this context, all universities with accredited Teacher Education programs must diagnose first year students to inform adjustments to their programs and design trajectories that enable students to achieve the goals described in their graduation profiles.

This situation naturally prompts questions such as: What characteristics of students must be determined before they begin their education to become teachers? Which of them would benefit from follow-up measures? What type of instruments would be useful to perform the necessary diagnoses? The answers to these questions should be aligned with the design of the teacher education program offered, since the literature describes how they all emphasize different elements. Research shows that successful educational systems provide education focused on discipline-specific and teaching knowledge, stressing the importance of teacher practices (Mourshed, Chijioke, & Barber, 2010; Musset, 2010) and striving to improve teachers' effectiveness in achieving learning goals during the first years of their professional life (Boyd, Grossman, Lankford, Loeb, & Wyckoff, 2009; Ortúzar, Flores, Milesi, & Cox, 2009).

However, several studies have highlighted the need to complement this approach with affective elements, which include pre-service teachers' belief systems (Hannula, 2012; Philipp, 2007; Schoenfeld & Kilpatrick, 2008; Thompson, 1992). The present study is based on the notion that beliefs are strongly influenced by perceptions, personal traits, and the decisions made throughout an individual's life. In turn, and in a cyclical manner, these beliefs are said to affect the decisions and actions of individuals, as well as how they interpret their experiences

(Bandura, 1986; Nisbett & Ross, 1980; Pajares, 1992). More specifically, beliefs play a major role in teachers' work, given their effect on the teaching-learning process: they generate a system that operates as a filter, structuring teachers' knowledge, perceptions, and decisions during their mathematics lessons (Freeman, 1989; Pajares, 1992; Richardson, 1996; Schmeisser, Krauss, Bruckmaier, Ufer, & Blum, 2013; Voss, Kleickmann, Kunter, & Hachfeld, 2013).

Based on this, we identified two relevant aspects to consider when applying diagnosis and follow-up mechanisms in any PTE program: the disciplinary contents to be taught and teachers' beliefs about the discipline and its teaching-learning process. In consequence, this study is aimed at answering the following research question: What do pre-service primary school teachers believe and know about school mathematics, its teaching, and its learning at the beginning of their education process? To answer this question, we employed an instrument focused on diagnosing pre-service primary school teachers' knowledge of school mathematics (which are imparted from 1st to 6th grade) as well as their beliefs about the discipline, its learning, and its teaching at the start of their professional training. This process enables us to present a characterization of both elements and explore some preliminary associations between them.

Conceptual framework

Several models have been devised to establish the domains of knowledge and practice that individuals should develop to become mathematics teachers and generate learning in their students (Font, Breda, Giacomone, & Godino, 2018). For instance, empirical research enabled Ball, Thames & Phelps (2008) to define Mathematics Knowledge for Teaching (MKT), which adopts and organizes the dimensions advanced by Shulman (1987), highlighting the need to attain specialized knowledge of the discipline in order to teach it. Ma (2010) suggests that a mathematics teacher requires in-depth knowledge of school mathematics, which must be based on an adequate command of mathematical connections. For their part, Carrillo, Contreras, and Flores (2013) complemented the MKT model by introducing individual belief systems, since they influence people's knowledge and practices. In the following section, we detail what we regard as Mathematical School Knowledge (MSK) as well as beliefs about mathematics, its teaching, and its learning, given that these two dimensions operate as axes that organize our work.

Mathematical School knowledge

The curricular framework establishes what society expects to achieve through its new generations; therefore, it is composed of dynamic parts associated with the development goals set (Cox, 2011; Kerr, 2002). In this regard, the skills and knowledge included in the school curriculum result from a secular demand for more capable citizens, who are required to engage in abstract thought, think systematically, experiment, solve problems, handle uncertainty, and adapt to change (Cox, 2011; Kerr, 2002; Ministerio de Educación de Chile, Mineduc, 2009). For this reason, over the last 10 years, several countries have modified their curricula in order to improve the quality of their educational systems and thus contribute to the development of their societies and the world (Hemmi, Lepik, & Viholainen, 2013). Chile was no exception: it adjusted the mathematics curriculum four times over the last 25 years (in 1996, 2002, 2009, and 2012). These modifications have foregrounded skills and attitudes (Cox, 2011). In this regard, the Ministry of Education (Mineduc) indicates that one of the main curricular innovations was to replace fundamental aims and minimum obligatory contents with learning goals, since they "establish a more explicit association between skills, knowledge, and attitudes, while also clearly and specifically conveying what students must learn" (Mineduc, 2012, p. 12). Thus, Chile evolved from providing a general specification of the learning goals that students were expected to achieve —detailed through a list of topics and contents that schools were obliged to cover— to delineating capabilities for completing tasks and solving problems with accuracy and adaptability, using mathematical knowledge.

As for the structure of mathematical knowledge as such, some differences can be observed between the latest two curricular frameworks. Indeed, in the official curriculum introduced by Mineduc in 2002, "the aims and contents presented are grouped around four thematic axes: numbers, arithmetic operations, forms and space, and problem-solving" (Mineduc, 2002, p. 82). In contrast, the current curricular framework comprises the following sets of mathematical contents: numbers and operations, patterns and algebra, geometry, measurements, and data and probabilities (Mineduc, 2012). This reveals two relevant differences: first, problem-solving is no longer a thematic axis and becomes a cross-curricular skill for developing mathematical thinking; second, there are two separate axes for concepts related to patterns and algebra and data and probabilities, two key topics in secondary education.

In addition, mathematical activity, apart from involving knowledge, also concerns skills. Indeed, international evaluations such as PISA and TIMSS focus on measuring both aspects. Specifically, the skill levels measured by TIMSS (Grønmo, Lindquist, Arora, & Mullis, 2013) appear to be more consistent with instruments aimed at determining how much a person knows relative to a prescribed curriculum, rather than determining the degree to which a person can perform certain processes involving mathematics, which is the case of the competence-based framework used in PISA (Organisation for Economic Co-operation and Development, OCDE, 2016). Therefore, this study considers the skill domains developed by TIMSS (International Association for the Evaluation of Educational Achievement, IEA, 2013): Knowing, Applying, and Reasoning. The first domain covers the facts, concepts, and procedures that students must know, while the second focuses on students' ability to apply said knowledge when solving problems. The third domain goes beyond the resolution of routine problems and comprises unknown situations, complex contexts, and multi-stage problems.

These two approaches to characterizing the mathematical school knowledge of people initiating their professional education as teachers, taken together and combined, make it possible to generate an instrument that simultaneously captures students' mathematical knowledge and capabilities. Also, with a view to orienting public policy, a curricular structure composed of thematic axes and skill levels makes it possible to inform both the school system and the great majority of teacher education programs, which means that the results of this study can be used more effectively.

Beliefs

We understand that beliefs are not just a verbalization of what one believes, but also the willingness to behave in a given manner (Wilson & Cooney, 2002). We also consider that beliefs do not operate in isolation: in the literature, belief systems are used as a metaphor to represent a possible structure of an individual's beliefs; that is, understandings and premises about the world, perceived as true by the person who holds them, which involve personal, cognitive, and affective codes and which predispose individuals to act in certain ways (Lebrija, Flores, & Trejos, 2010; Lester, Garofalo, & Kroll, 1989).

When designing this study, we strove to find a classification that would enable us to organize the participants' beliefs and help us determine which items to include in the instrument; in addition, we examined several views of mathematics, its teaching, and its learning as a way of giving the items on beliefs a sense of direction.

In this regard, research on mathematical education has defined three categories within belief systems, which make it possible to describe several aspects of the mathematical activity taking place in the classroom (Op't Eynde, De Corte, & Verschaffel, 2002): beliefs about mathematical education, beliefs about oneself, and beliefs about the social context. Based on this study, and considering other research on the beliefs of active and pre-service teachers, we devised the following categorization:

- Beliefs about the nature of mathematics. Beliefs about the inherent characteristics of mathematics as a discipline.

- Beliefs about the teaching-learning process of mathematics (Donoso, Rico, & Castro, 2016). Teaching methodologies, students' and teachers' characteristics, the social context, and the social dynamics of the classroom, among other elements.
- Beliefs about oneself. Students' attitudes to mathematics and how these influence the learning process (Palacios, Arias, & Arias, 2014).

The mathematical experiences and knowledge that individuals accumulate throughout their life define their belief systems regarding mathematics as well as its teaching and learning, all of which gives them some "perspective" in this respect. Felbrich, Kaiser, & Schmotz (2012) distinguish a constructivist and a transmissive perspective, which are related to dynamic and static views of mathematics respectively (Kaiser & Maaß, 2007; Köller, Baumert, & Neubrand, 2000). The transmissive perspective establishes that knowledge should be taught directly, focusing on definitions, theorems, and algorithms that students will be required to use. Class structure is rigid and teacher-centric, with students being receivers of knowledge and successful mathematical learning depending on student characteristics, that is, intrinsic capabilities or skills for learning (Köller et al., 2000; Staub & Stern, 2002). In contrast, from the constructivist perspective, the lesson is regarded as a process of interaction leading to the development of knowledge, with students and their experience playing a major role in the teaching and learning of mathematics. In this view, mathematical learning depends on the characteristics of the activities proposed, interaction, and the role played by the teacher and students (Köller et al., 2000; Staub & Stern, 2002).

Methodology

In this study, we constructed the instrument, validated it, and analyzed the results it yielded with an exploratory-descriptive design, using mixed (qualitative and quantitative) data analysis methodologies (Creswell, 2012). We chose this approach because the larger project to which this article belongs, as a whole, is aimed at constructing an instrument considering two validation levels (content and construct) from the design phase onward, to later describe some characteristics of the target population. In order to do this, the initial phase of the study focused on the construction of the first version of the instrument. This involved several methodologies, from the generation and analysis of an in-depth interview —to validate belief constructs within the context of Chile's PTE— to expert judgment to test the items developed to evaluate MSK (Martínez et al., 2018).

In the second phase, we worked to validate the instrument and characterize first-year students enrolled in primary school teacher education programs. It is the latter aspect that is described in this article; therefore, the following section details the definition of the sample used as well as the instrument administered and the analyses performed.

Participants

The Teacher Professional Development System (Law 20.903) establishes that students must score at least 500 points on the University Selection Test (Prueba de Selección Universitaria, PSU) (in the Language and Mathematics parts) to enroll in Teacher Education programs. This score is to increase gradually over time. This means that universities are required to secure accreditation; therefore, we defined the sample considering the universities that met these characteristics (Patton, 2001). To specify the variables used and stratify the sample, we took into account the strong segregation of the whole Chilean educational system (Valenzuela, Bellei, & De Los Ríos, 2014), a matter that is reflected in the PSU score distribution. First, individual belief systems about mathematics, its learning, and its teaching are chiefly generated during people's school education (Pajares, 1992; Schmeisser et al., 2013; Thompson, 1992; Voss et al., 2013). Since students who enroll in Teacher Education programs have a variety of school trajectories and experiences due to segregation, they can have multiple belief systems in this regard, as well as different levels of knowledge about school mathematics. Second, it can be observed that the type of school attended by students who enroll in Primary School Teacher Education programs in universities belonging to the Council of Presidents of Chilean Universities (Consejo de Rectores de Universidades Chilenas, CRUCH) correlates with the distribution of PSU scores (Valenzuela et al., 2014). In this context, the variables selected are the following:

- Average PSU score (Mathematics and Language): differentiated by quartiles within the range of PSU scores.
- Type of university: "traditional" (affiliated with the CRUCH) and "non-traditional".

Based on the above criteria, we defined a sample of 400 first-year students enrolled in Elementary School Teacher Education programs in 2017, which represented about 25% of the total enrollment figure. The distribution of the variables defined was proportional; that is, according to the number of students enrolled in Teacher Education programs by quartile and type of university.

Table 1 shows the number of students who had to answer the questionnaire to achieve $n = 400$, according to the proportional distribution. The number of students who actually completed the questionnaire is in brackets.

Table 1. Average PSU score (Mathematics and Language) of students enrolled in 2017.

		500 - 540	541 - 553	554 - 570	More than 571
Type of university	Traditional	62 (68)	48 (58)	110 (94)	98 (138)
	Non-traditional	22 (34)	28 (24)	17 (48)	14 (47)

Note: Proportional number of participants for the planned sample (n = 400) and number of actual participants (n = 511), by quartile and type of university.

Source: Own work.

The final sample was composed of 511 students, which represents approximately one third of the nationwide enrollment in 2017. In terms of institutions, the participating students attend 14 universities, six located in the Metropolitan Region and eight in other regions of the country. In this context, 71.61% of the questionnaires were answered by students attending traditional (CRUCH) universities and 28.39% by students enrolled in non-traditional universities.

Instrument

The instrument comprised two sections: Beliefs about mathematics, its teaching, and its learning and Mathematical School Knowledge (MSK). The first section was composed of 47 Likert scale items, where respondents were asked to select their level of agreement from 1 (*strongly disagree*) and 4 (*strongly agree*). These items were organized as shown in Table 2, considering the literature and its validation in Chile.

Table 2. Distribution of items for evaluating beliefs by category and subcategory

Category	Subcategory	Nº of items
Teaching and learning	Learning	5
	Teaching	9
	Classroom social dynamics	1
Expectations and achievements	Conditions for achievement	5
	Self-perception	5
	Anxiety/attitude	6
	Family	3
Mathematics	Nature of mathematics	2
	Nature of mathematical thinking	4
	Nature of mathematical actions and their relationship with the world	4
	Nature of mathematics and its relationship with students	3

Source: Own work.

The second section comprised items for evaluating the MSK of students who enroll in Primary School Teacher Education programs, which were arranged according to the mathematical knowledge present in the learning goals established in the elementary education curriculum currently in force (Mineduc, 2012) and according to the cognitive domains defined by TIMMS, considering the skills present in each. In total, 40 item were distributed after the validation process, as shown in Table 3.

Table 3. Final distribution of items by knowledge domain and skill

		Skill			
		Knowing (K)	Applying (A)	Reasoning (R)	Total
Knowledge	Numbers and operations (N)	11	8	1	20
	Measurement (M)	2	1	1	4
	Geometry (G)	3	4	0	7
	Algebra (A)	0	2	2	4
	Data and probabilities (D)	0	4	1	5
	Total	16	19	5	40

Source: Own work.

Analysis

First, during the prior validation stage, the items on beliefs underwent an exploratory factor analysis, considering several methods for determining what number of factors would adequately reflect the clustering of the items. We employed Kaiser's method, which suggested that we keep 13 factors, but the decay graph displayed a sharp slope change after the eighth factor. In contrast, the MAP (Minimum Average Partial) method advanced by Velicer

(Velicer, 1976) suggested using seven factors. After these analyses, we performed another factor analysis, now with twelve and 8 factors. In the first case, the last four of 12 factors displayed low consistency and reliability. In the eight-factor analysis, which managed to explain 79.68% of the variance, one of the factors generated a new one, associated with the role of family, which resulted in a final nine-factor set that explained 82.80% of the variance. Afterward, the Bartlett method determined that the correlations among these factors were low (less than 0.319); therefore, we chose to describe beliefs using clustering techniques that would enable us to characterize students. This will be presented in detail in the following section.

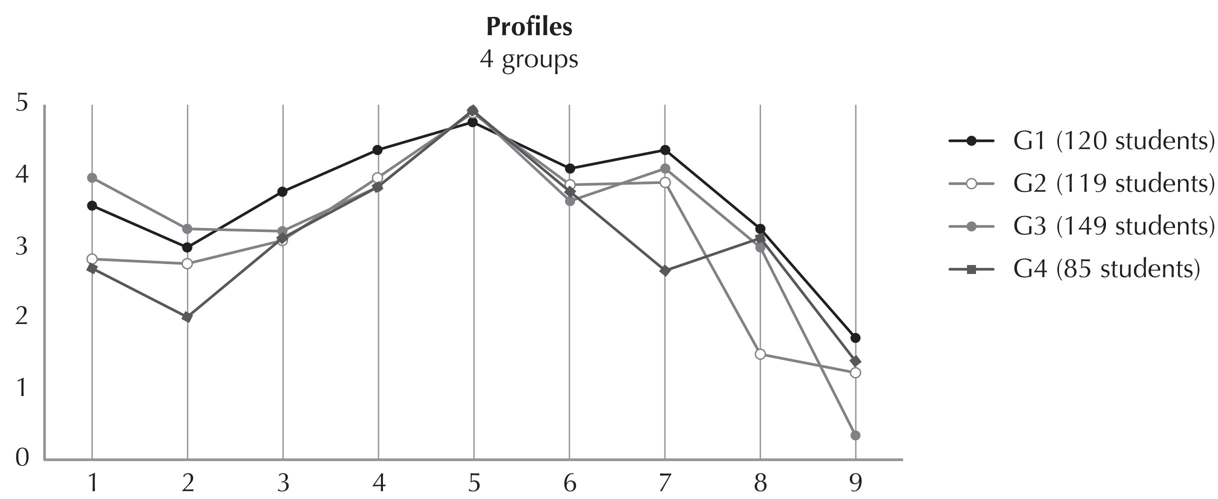
As for MSK, and as part of the validation of the instrument, we performed an exploratory factor analysis (EFA) using the matrix of polychoric correlations, with the Kaiser criterion suggesting 16 factors. Considering the overestimation of this method, we took into account the decay graph and the MAP method, which suggested six factors that explained 67.06% of the variance. The full scale displayed an internal consistency level that ranged from 0.8 as underestimation to 0.88 as overestimation of reliability, with Cronbach's alpha = 0.82. The assumptions of one-dimensionality and local independence being met by the prior factor analyses, we applied item response theory (IRT) with a 2PL model to the 40 items grouped into the six factors, determining the parameters of difficulty, discrimination, and latent skill of the Teacher Education students who completed the questionnaire. Only seven of the 40 items displayed discrimination levels under 1, which indicates that all items have moderate to very high discrimination levels. Lastly, we conducted a descriptive analysis considering the content axes and skills, in order to describe the participants' performance considering both variables.

Lastly, and in order to link students' beliefs with their MSK, we analyzed the correlation between the latent classes related to beliefs and the factors related to MSK obtained in the validation phases, whose results are presented below.

Characterization of beliefs and MSK

Based on the administration of the instrument and its later construct validation for both sections of the study (beliefs and MSK), we present the characterization of the participants' results in each area, along with the associations between them.

The characterization of the beliefs reported by the participants made it possible to divide the sample into four groups after a clustering process informed by the nine factors obtained. Graph 1 presents the centroids of each group by factor, revealing that students possess a common core of beliefs.



Graph 1. Centroids of groups 1 to 4, by factor of Likert scale items.
Source: Own work.

As previously noted, the groups of students cannot be "ordered" in terms of their views on mathematics and its teaching-learning (Felbrich et al., 2012; Kaiser & Maaß, 2007; Köller et al., 2000), since the participants are characterized by a variety of heterogeneous beliefs, with the categories and subcategories defined here being spread equally among them. Nevertheless, said groups can indeed be differentiated, as certain elements that characterize and distinguish them from the rest can be clearly identified.

- Group 1: participants have a more positive view of mathematics than the rest and think that the discipline is creative. Likewise, they consider that good teachers are creative and challenging, and think that discussions with others are important for learning.
- Group 2: these are moderate participants, since they regard mathematics as a rigid discipline. They perceive less family support than the other groups regarding mathematics learning.
- Group 3: these participants have a more positive view of themselves with respect to learning. In their opinion, a good teacher proposes simple and clear activities and does not need to be creative or fun.
- Group 4: students who tend to agree with traditional statements about mathematics and who do not think that it can be learned in a dialogical and argumentative context.

Since all these groups comprise students with a variety of PSU scores —which is strongly associated with segregation in Chilean education (Valenzuela et al., 2014)—, it is relevant to determine how students are distributed with respect to this variable. First, according to their distribution by PSU score quartile, we observed a significant association[1]. This means that the beliefs about mathematics and its teaching-learning held by students who enroll in Primary School Teacher Education programs are strongly linked to their performance and thus with their learning opportunities as school students.

Second, we also observed (Tabla 4) that the universities with the highest admission scores have a larger percentage of group 4 students, who are less prevalent in the rest of the universities. This suggests that the type of teaching that produced better performance consisted in mathematical activity that students regard as more transmissive. Also, the larger percentage of group 1 students in the first enrollment quartile suggests that greater learning needs are linked to a stronger emphasis on presenting mathematics in less traditional ways. Lastly, average-performance students (quartiles 2 and 3) are, on the whole, similarly distributed among groups 1 to 3, which reflects some degree of heterogeneity in their perception of mathematics and its learning and teaching.

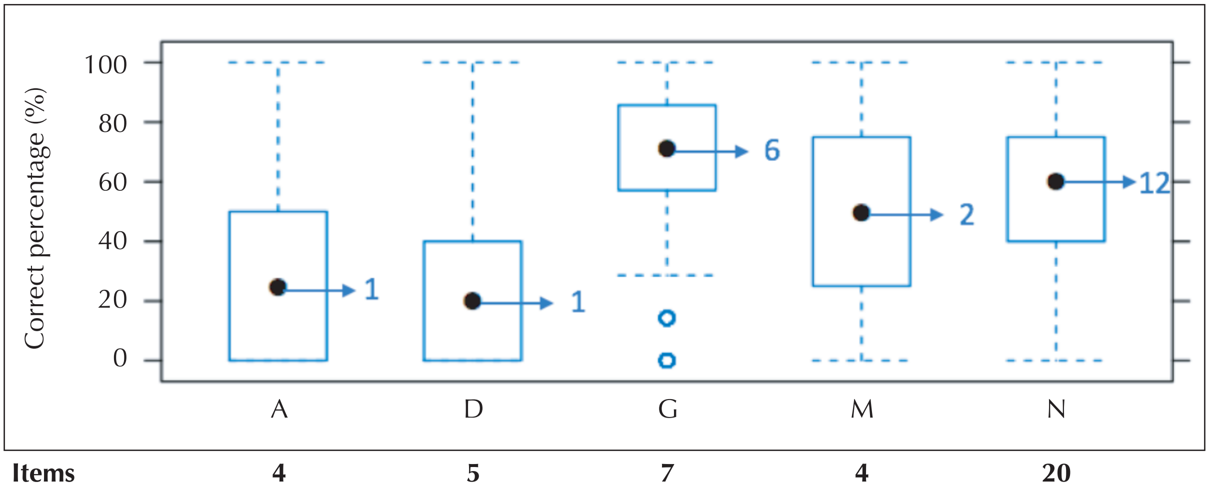
Table 4. Contingency table: belief group and PSU score

	Group				
Quartile	1	2	3	4	Total
500 - 540	35	23	23	13	94
541 - 553	27	26	22	3	78
554 - 570	36	37	38	18	129
571+	22	33	66	51	172
Total	120	119	149	85	473

Source: Own work.

1. The statistical tests performed yielded the following values: ($\chi2(9, N = 473) = 52.331, p < 0.001$).

Regarding the MSK shown by the respondents, their command of mathematical contents can be characterized by curricular axis and by difficulty level, taking into account the cognitive domains associated with each item. Graph 2 shows the percentage of correct answers by area of mathematical content.



Graph 2. Percentage of correct answers in the following axes: Algebra (A), Data and probabilities (D), Geometry (G), Measurement (M), and Numbers and operations (N).

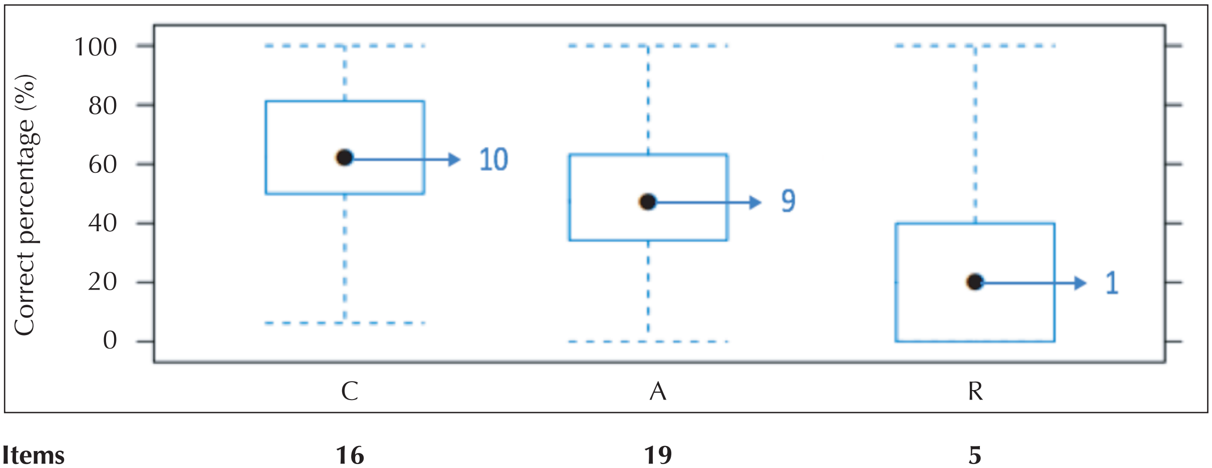
Note: The numbers shown in the graph represent the quantity of items answered correctly by 50% of the students (median) on each content axis. The number outside the graph, at the bottom, represent the total number of items per axis.

Source: Own work.

First, these results show that at least one student answered all items in all content axes correctly and that at least one student answered all items incorrectly. This reflects the wide variability among students in terms of MSK, especially with regard to the contents established by the curriculum from 1st to 6th grade. More specifically, our findings show that 50% of the participants answered 25% of the algebra items correctly (one item); 20% of the Data and probabilities items correctly (one item); 71% of the Geometry items correctly (five items); 50% of the Measurement items correctly (two items); and 60% of the Numbers and operations items correctly (12 items).

Regarding item difficulty, Geometry proved to be the least difficult axis, with 75% of the students answering at least 60% of the items correctly. Similarly, the Numbers and operations axis emerged as the second least difficult, as the same percentage of students answered at least 40% of the items correctly. As for the Measurement axis, the respondents performed at an intermediate level, as 50% of them answered between 30% and 70% of the items correctly, which reflects high result dispersion. Lastly, Algebra and Data and probabilities proved to be the most difficult axes. In the former area, only 25% of the respondents answered 50% of the associated items correctly; in the latter, the same percentage of students answered 60% of the items correctly.

Graph 3 shows the participants' results according to their median performance on each item.



Graph 3. Percentage of correct answers by skill: Knowing (C), Applying (A), and Reasoning (R).
Source: Own work.

These results show that, in each of the three skills, there is at least one student who answers all items correctly, while only in Applying and Reasoning is there one who does not answer any items correctly. As in the case of mathematical knowledge, these findings illustrate the great variability among students in terms of how they approach questions of several levels of complexity. In addition, the data reveal a decrease in performance as complexity increases, with a major difference being observed between Applying and Reasoning. This is consistent with the theoretical degree of complexity of these three skills.

Specifically, 50% of the students below the median answered about 63% of the Knowing items correctly (ten out of 16); 47% of the Applying items correctly (nine out of 19); and 20% of the Reasoning items correctly (one out of five). In addition, it is clear that the respondents performed better in the least cognitively taxing skill (Knowing), as 75% of them answered at least half the items correctly (eight out of 16). The same percentage of students answered only seven of the 19 Applying items correctly (37%); as for Reasoning, only 25% of students answered three of the five items correctly. This performance shows that, despite the smaller number of items, students find it hard to answer items that pose greater cognitive demands.

All in all, results show that respondents have characteristic and differentiable belief systems, with a common core that is strongly related with their PSU performance. In addition, they display greater mathematical knowledge in the content axes that are more prevalent in the Chilean curriculum, such as Numbers and Operations and Geometry. Similarly, they display a poorer disciplinary command of elementary education contents that have recently been introduced into the curriculum, such as Algebra and Data and probabilities. Finally, students' performance decreases as the mathematical complexity of the topics measured with the instrument increase.

Associations between beliefs and MSK

Given the independent characterizations of beliefs and MSK displayed by the respondents, it is interesting to explore some potential connections between these dimensions. To do this, and based on the validation of the instrument, we first conducted factor and latent class analyses for MSK and beliefs, respectively. These analyses yielded nine factors grouping the beliefs answers and six factors grouping the MSK answers. Both are presented below.

Beliefs factors	MSK factors
B1: I have a talent for mathematics, I believe in myself. B2: Mathematics is not a rigid discipline that only consists in formulas and memorization. B3: Mathematics is important to me. B4: Mathematics presents open and creative solutions. B5: Teachers must be interested in getting their students to learn. B6: Some people have an innate facility for mathematics. B7: Having discussions with others is a learning strategy. B8: Family support is very important. B9: A good teacher must be creative and fun, not simple.	MSK1: Composed of 12 items from the Numbers and operations axis involving the Knowing skill, plus one item from the Geometry axis. MSK2: Composed of seven items from the Numbers and operations axis, five of which belong to Applying and two to Knowing, plus one item from the Measurement axis. MSK3: Composed of eight items, four from the Algebra axis involving Applying and Reasoning, three items from the Measurement axis involving Knowing and Reasoning, and one Numbers item involving Reasoning. MSK4: Composed of seven items, mostly from the Geometry axis and associated with Knowing and Applying, plus one Measurement item. MSK5: Composed of six items, five from the Data and probabilities axis involving Applying and Reasoning, plus one Algebra item. MSK6: Composed of four Numbers items involving Knowing and Applying.

Afterward, and to analyze the relationship among these factors, we studied the correlations between the latent variables of both dimensions. Table 5 shows the Pearson correlations among the nine latent variables associated with beliefs (B1 to B9) and the six latent variables associated with content (MSK1 to MSK6).

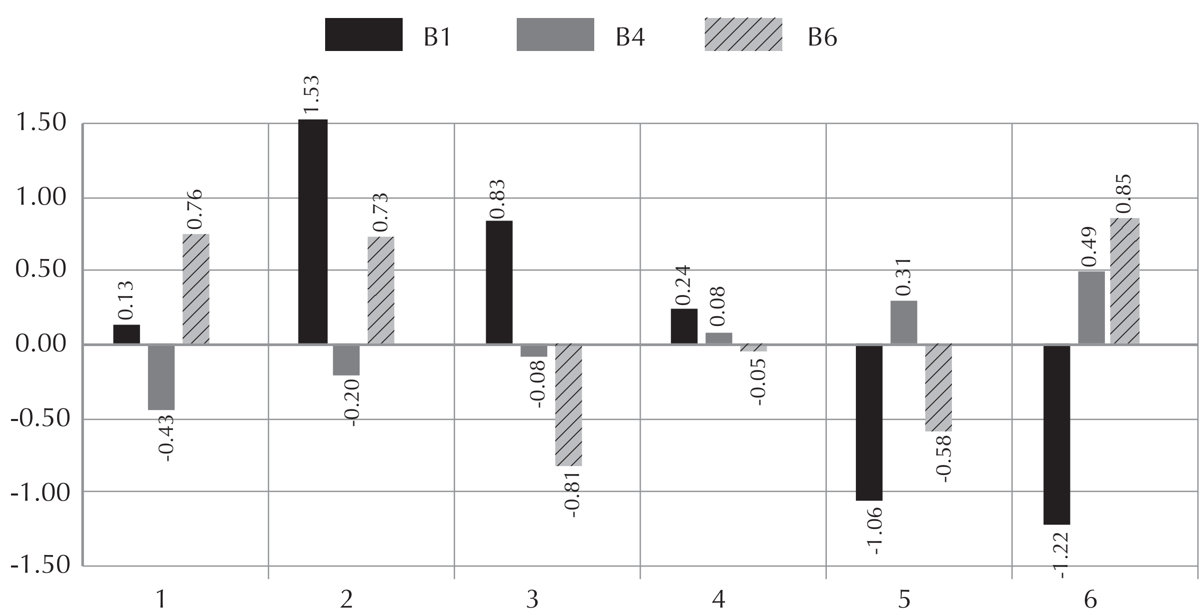
Table 5. Pearson correlations among latent variables

	B1	B2	B3	B4	B5	B6	B7	B8	B9
MSK1	**0,36****	0,03	0,08	**-0,13***	0,07	**0,12***	-0,04	0,09	-0,07
MSK2	**0,19****	0,09	-0,02	**-0,14****	0,07	**0,13***	0,03	0,07	-0,02
MSK3	**0,09**	0,05	0,02	**-0,12***	0,03	**0,02**	-0,03	0,03	-0,02
MSK4	**0,23****	0,11*	0,03	**-0,08**	0,10	**0,12***	-0,07	0,09	0,02
MSK5	**0,19***	0,09	0,06	**-0,03**	-0,02	**-0,13***	-0,02	0,02	-0,10
MSK6	**0,30****	0,12*	-0,01	**-0,11***	0,02	**0,10**	-0,08	0,11*	-0,13*

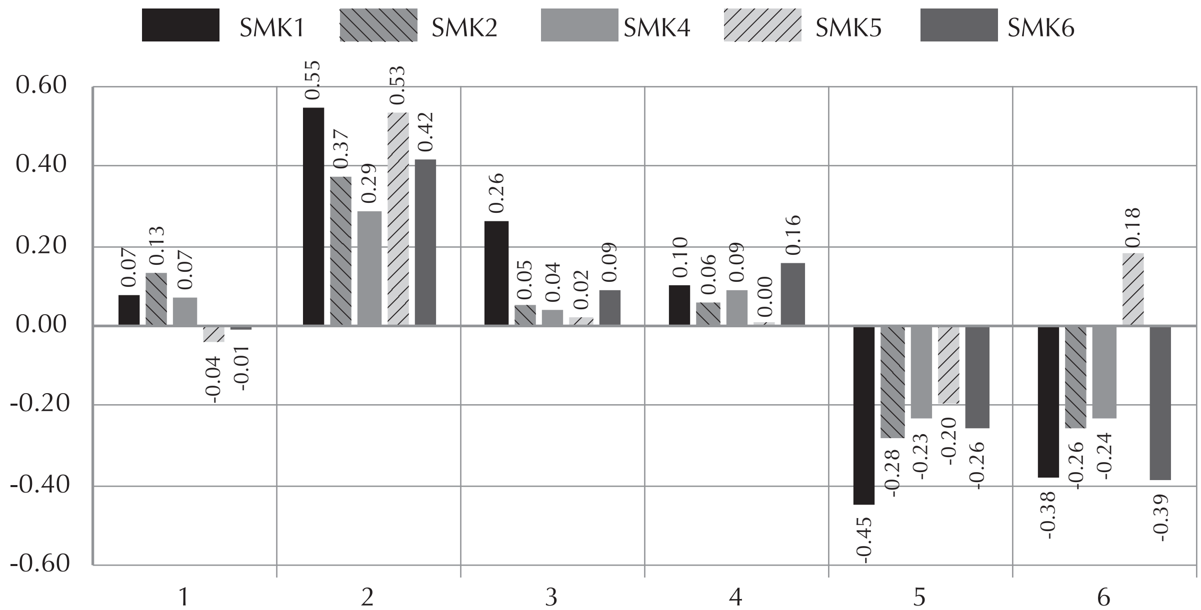
*Note: Correlations marked * and ** are intermediate and high, respectively.*

Source: Own work.

It can be observed that only three of the nine latent belief variables display most of the correlations with MSK. Specifically, B1 has a high or intermediate correlation with five of the six latent MSK variables. B4 has an intermediate or high inverse correlation with four of the six latent MSK variables. Finally, variable B6 displays an intermediate correlation with four of the six latent content variables. In this context, we performed a cluster analysis process according to the latent variables B1, B4, and B6, which yielded six differentiated groups according to Ward's method. Graph 4 analyzes the averages of the latent beliefs variables and Graph 5 shows those of the MSK variables, for all six groups.



Graph 4. Averages of latent beliefs variables by group.
Source: Own work.



Graph 5. Averages of latent content variables by group 2.
Source: Own work.

Based on our analysis of these graphs, it is relevant to highlight certain aspects that might shed light on possible associations between students' beliefs and their MSK. For instance, some "opposite" groups emerge when we consider the averages of latent variables related to both beliefs and MSK, such as groups 2 and 5. Similarly, it can be observed that groups 5 and 6 share a very poor self-perception of mathematical talent (B1) and a high perception of mathematical flexibility (B4), but differ in their perception of innate talent (B6). In contrast, these groups behave similarly in MSK variables, except for the factor focused on Data and probabilities (MSK5).

As for beliefs variables (Graph 4), group 1 views mathematics as a rigid discipline that requires innate talent, with respondents being indifferent to their own talent. In contrast, group 2 has a very positive perception of their own mathematical talent, with respondents considering that the discipline is moderately rigid and believing that innate facility exists. Respondents in the third group have a positive perception of their mathematical talent, but do not believe that talent is innate; rather, they think that it can be learned and developed. The fourth group appears to be indifferent to everything. Respondents placed in group 5 seem to have a very negative perception of their own mathematical talent and regard mathematics as a creative discipline, but do not think that innate mathematical talent exists. Lastly, group 6 is composed of students who do not think that they have mathematical talent or facilities, but believe that mathematics is creative and innate.

As for latent MSK variables (Graph 5), it is interesting to note that MSK5 stands out due to its moderately high value in group 6, the second highest after that of group 2, where this factor displays the second lowest value. It should be noted that, even though group 6 respondents believe in creative mathematics (B4) and those in group 1 think that it is not creative, they have a similar belief in an innate facility for mathematics. Since factor MSK5 is associated with Data and probabilities items, a connection can be observed between the belief that mathematics is creative and the resolution of problems in this axis (considered to be difficult), although this is not visible in the statistical correlation values.

2. MSK3 is omitted from the analysis since it only correlates with two of the three latent beliefs variables selected (see Table 5).

In general, it is interesting to note that respondents from groups 5 and 6 have a negative perception of themselves (unlike those from groups 1, 2, 3, and 4) and have the lowest averages in the latent variables related to MSK (except for MSK5 in group 6), while also being the most appreciative of creativity. These elements are starting points for a deeper examination of the links between the knowledge and the belief systems of pre-service teachers.

Conclusions

The requirement for teacher education institutions to conduct a diagnostic evaluation at the start of their programs, in accordance with the new Teacher Professional Development System (Law 20.903), will produce relevant and valid information for making decisions about all cohorts enrolling in them. Said information will be useful for adapting and designing trajectories that will allow students to achieve the goals described in the graduation profiles of their teacher education programs. In this regard, the results included here represent an opportunity for the Education faculties that train Elementary Education teachers to rethink the mathematical learning opportunities prescribed in the curricular trajectories, based on the characterization of MSK from 1st to 6th grade and students' beliefs about mathematics teaching and learning. In this context, we stress the importance of designing evaluations that enable us to know students in areas other than those examined by known measures. This encouraged us to define MSK as an aspect different from those evaluated by the university selection tests and which lays the groundwork for the specialized mathematical knowledge that pre-service teachers must develop.

Likewise, studying beliefs provides highly relevant information within the context of pre-service teacher education, especially regarding students who are starting their careers. As specified earlier, beliefs constitute a disposition to act (Lebrija et al., 2010; Lester et al., 1989; Wilson & Cooney, 2002) and their construction throughout a person's life can determine his/her views about mathematics, its teaching, and its learning. Our analysis of the beliefs held by Primary School Teacher Education students at the start of their training, informed by clustering analysis methods, made it possible to characterize the beliefs of these students about school mathematics, its learning, and its teaching. Four groups were identified, with an overall shared behavior in some factors and differences in other cases, the latter being what enabled us to characterize these groups. In this regard, interestingly, the clusters are clearly distributed depending on students' university admission scores. This is evidence that students' beliefs at the start of their teacher education are heterogeneous and that differences between universities exist and can be characterized; therefore, it is worthwhile for Chilean higher education institutions to strive to determine these beliefs and provide learning experiences in line with their students' background.

Regarding the MSK of first-year students of Elementary School Teacher Education programs, results show a performance level consistent with the current or past curricular presence of the content axes in this educational cycle: the topics with the smallest percentage of correct answers are Algebra and Data and probabilities, as opposed to Geometry and Numbers and operations. In this regard, it is relevant to consider that the participants who took the questionnaire had no opportunities in their primary education to learn the skills and contents belonging to the axes where they performed poorly, since they were only introduced to the curricular framework in 2012. This is not true of Geometry and Numbers and operations, because both the current and the past curricular frameworks contain a large percentage of knowledge related to numbers, operations, and geometry. As for skills, it is clear that the mathematical reasonings that require employing and applying mathematical information are the most complex for students, regardless of the thematic axis. Similarly, students perform better on items involving skills that require fewer complex cognitive aspects. Both findings suggest that learning activities performed in disciplinary and didactic classes should focus on topics which are less prevalent in the school curriculum and which are more cognitively demanding. Thus, students will be able to articulate mathematical information for decision-making, the formulation of conjectures, and problem-solving

When analyzing the initial associations between the latent beliefs variables and the factors of the MSK section, an especially attention-worthy correspondence is that students who perform better on the Numbers and operations axis have a better self-perception of their mathematical ability and find creativity less valuable. Given the widespread notion that mathematics is exact, requires mental speed, and is connected with numerical work (to the detriment of spatial or stochastic work), we consider that it may be natural to associate good performance on the Numbers axis and think that said capabilities determine "being good at math". This leads us to reflect on how mathematical activity is defined in school classrooms and how this perception connects with pre-service teacher education; therefore, it will be necessary to offer multiple opportunities for pre-service teachers to expand their conception of mathematics and what it means to be mathematically competent.

Lastly, based on the study conducted, we consider that it is essential to highlight the importance of the work being conducted in the domain of diagnostic evaluations in Chile. First, we think that it is necessary for public policy to encourage research aimed at creating instruments which complement each other, which make it possible to measure deep and specific knowledge about the teaching, and which are administered at the start of teachers' professional career. To do this, it will be necessary to generate multi-method strategies, that is, strategies that adopt a variety of forms but that operate together to extract multiple types of information; for example, questionnaires for assessing perceptions and knowledge, interviews for evaluating decision-making processes, performance simulations, and classroom observations, among others. Second, it must be pointed out that a process as costly and complex as developing evaluations of this type highlights the need for collaboration among universities in order to optimize the work involved, share findings, and engage in country-wide efforts that make it possible to achieve educational improvements delineated as a common aim.

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