ENGLISH VERSION

Probabilistic analysis of slope stability in completely decomposed granite residual soils

Evaluación probabilística de la estabilidad de taludes en suelos residuales de granito completamente descompuesto

W. Fernández *, S. Villalobos [1]*, R. King *

* Universidad Católica de la Santísima Concepción, Concepción. CHILE

Fecha de Recepción: 24/10/2017
Fecha de Aceptación: 06/03/2018
PAG 05-14

Abstract

The effects of physical and chemical weathering processes in completely decomposed granitic rocks strongly condition the shear strength parameters commonly used in soil mechanics, where soils have high variability. The technological progress and the joint use of deterministic and probabilistic methods allow for the incorporation of soil variability within the calculations. This paper presents a probabilistic slope stability analysis using a model of random variables to characterize the shear strength parameters. The Monte Carlo simulation is used for generating the values of the random variables, which also allows for the simultaneous evaluation of slope stability in terms of the factor of safety, probability of failure and reliability index. The results of this methodology tend to be more conservative, considering the risk of using average safety factors to evaluate stability when considering the variability of soil properties.

Keywords: *Residual soil, Monte Carlo simulation, slope stability, factor of safety, probability of failure, reliability index*

Resumen

Los efectos de los procesos de meteorización física y química que han sufrido las rocas graníticas completamente descompuestas condicionan fuertemente los parámetros de resistencia al corte utilizados comúnmente en la mecánica de suelos, siendo suelos que presentan una alta variabilidad. Con el avance de la tecnología, el uso conjunto de métodos determinísticos y probabilísticos permite la incorporación de la variabilidad del suelo dentro de los cálculos. En este artículo se presenta un análisis probabilístico de estabilidad de taludes utilizando un modelo de variables aleatorias para caracterizar los parámetros resistentes. Para la generación de valores de las variables aleatorias se utiliza la simulación de Monte Carlo la cual además permite la evaluación simultanea de la estabilidad del talud en términos de factor de seguridad, probabilidad de falla e índice de confiabilidad. Los resultados de esta metodología tienden a ser más conservadores, considerándose bastante riesgoso la utilización de factores de seguridad medios para evaluar la estabilidad cuando se considera la variabilidad de las propiedades del suelo.

Palabras clave: Suelo residual, simulación de Monte Carlo, estabilidad de taludes, factor de seguridad, probabilidad de falla, índice de confiabilidad

1. Introduction

Many civil engineering works require the execution of temporary and permanent slopes, for example, in excavations for buildings, open cuts for highways, soil reservoirs, embankments. Therefore, it is interesting to evaluate the slope stability for potential slip failures. The deterministic design method commonly used in geotechnical engineering is characterized by the high uncertainty of the variables considered. Soil properties vary from one location to the other and they can also change over time; consequently, the information obtained for one specific place will not guarantee the information for any other place. Based on laboratory test results, the uncertainty also arises from the calculation of the soil's shear strength property on site (cohesion and angle of internal friction). In these tests, the actual conditions of the soil cannot always be reproduced perfectly; for example, given the changes in the load system, perturbations during sampling, anisotropy, and pore pressure, among others (Tang et al., 1976). Therefore, in order to deal with this range of uncertainties, the deterministic method applies design safety factors, which may be partial or global, depending on the case.

Since 1975, probabilistic methods that are complementary to the deterministic method have been developed to evaluate the slope stability. The deterministic method is based on minimizing the factor of safety in a variety of potential failure surfaces, thus determining the surface of the minimum safety factor, known as the critical slip surface. Furthermore, a commonly used probabilistic method determines the reliability of the slope, based on the calculation of the probability of failure and the reliability index, corresponding to the critical slip surface. Additionally, the probabilistic analysis can be used for arbitrary slip surfaces, that is, considering different specific slip surfaces that are not associated to the minimum factor of safety or reliability index (Bhatacharya et al., 2003).

Several probabilistic methodologies that incorporate the uncertainties in the slope stability analysis have been developed, such as the first-order reliability method (FORM), the first-order second-moment (FOSM), the response surface methodology (RSM), and the Monte Carlo simulation (MCS), among others (Baecher and Christian, 2003). The latter has gained popularity due to its solidity and conceptual simplicity (Wang et al., 2010), which generates a large quantity of random variables, within a range and according to the probability distribution, for parameters considered in the analysis; afterwards, it evaluates the stability in terms of the probability of failure (Reale et al., 2015).

Based on laboratory and field data, this paper seeks to demonstrate the application of probabilistic techniques to evaluate the stability and potential failure of slopes in

[1] *Corresponding author:*

Departamento de Ingeniería Civil, Universidad Católica de la Santísima Concepción, Concepción, Chile.
E-mail: svillalobos.ic@gmail.com

completely weathered granite residual soils, particularly in the region of Concepción. Using a limit equilibrium method and explicitly incorporating the variability of the soil's shear strength properties based on the data generated by the Monte Carlo simulation, the slope stability is evaluated in a probabilistic form, thereby obtaining the probability of failure and the reliability index for different slope geometries.

2. Granite residual soil

Lately, the city of Concepción has been characterized by developing real estate and infrastructure projects in areas with residual soils, mainly formed by the decomposition of granite rocks. Several researchers have experimentally studied the shear strength properties and the slope behavior in these types of soils (Ruiz, 2002; Cabrera, 2007; Villalobos, 2011; Villalobos et al., 2013; Rodríguez, 2015; Flandes, 2017).

In residual soils, the variations in the mineralogy and grain size of the bedrock, the chemical weathering process, physical disintegration, hydrothermal alteration and leaching produce heterogeneous soils, which is an important source of uncertainty for calculating the shear strength properties and identifying potential failure mechanisms in the slopes (El-Ramly et al., 2005).

Figure 1 shows the existing geological unit in the city of Concepción, which corresponds to a highly weathered intrusive igneous rock forming part of the Coastal Batholith. This Paleozoic granite rock is between 250 and 570 million years old and it is the result of the tectonic activity caused by the subduction of the Nazca Plate under the South American Plate. The weathering has destroyed the union between the mineral grains and it has transformed a rock with high mechanical strength into disintegrated blocks of residual soil, a gravel locally known as Maicillo. This type of soil is very complex to analyze, because it is hard to determine whether it will behave as a rock or a soil, or a combination of both (Villalobos et al., 2013).

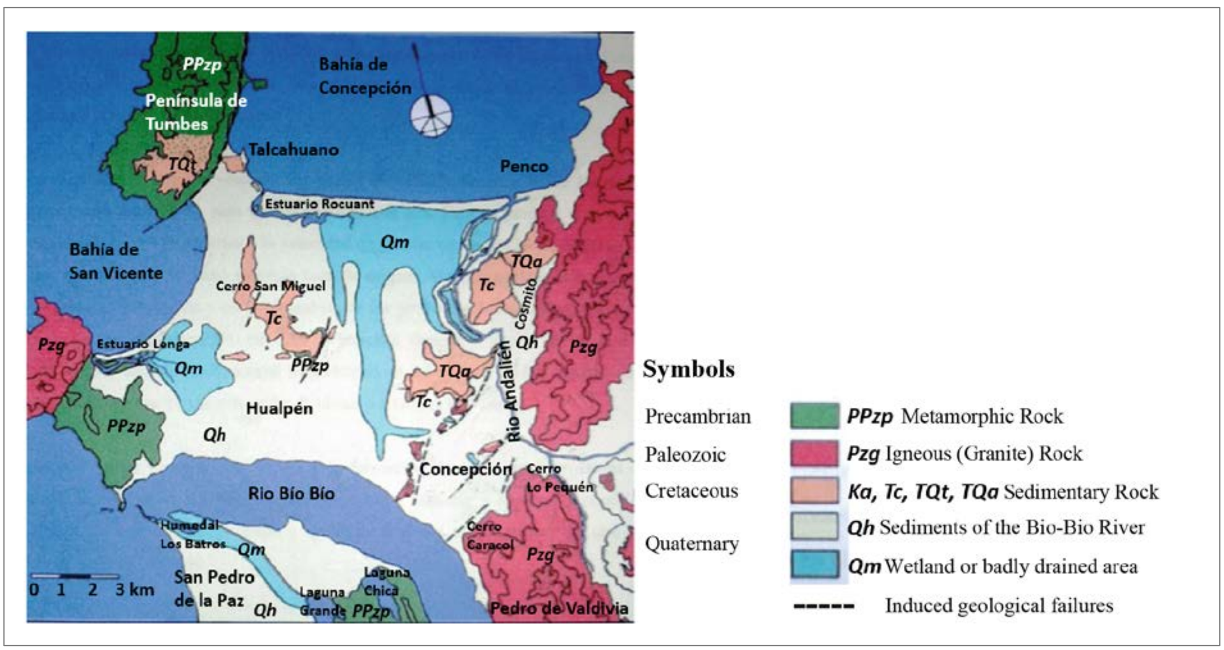

Figure 1. *General geological map of Concepción (Poblete and Dobry, 1968)*

2.1 Properties' variability in residual soils

Residual soils are formed by weathering of the underlying primary rock material and their overall chemistry is very similar to the original rock material (Brady and Weil, 1996). The weathering of the original rock material may occur through physical weathering (disintegration), chemical weathering (decomposition) or chemical transformation. Therefore, the properties of these soils can drastically change due to the variation of parent materials and different weathering forms (Zhai et al., 2016).

The geotechnical properties of residual soils vary spatially, even within the same stratum. However, most geotechnical analyses adopt a deterministic approach based on average soil parameters applied to each stratum. The conventional tool to deal with soil heterogeneity in on-site conditions is the application of safety factors implemented according to the local experience and the engineering judgement (Elkateb et al., 2003). However, it has been recognized that the factor of safety is not a consistent measure to evaluate the slope risks, since slopes with the same safety factor can exhibit different risk levels, depending on the variability of the soil's properties (Li and Lumb, 1987). Consequently, in the last years, several researches have been conducted with the aim of validating the probabilistic slope stability analysis, by systematically addressing the uncertainties of the soil properties (Alonso, 1976; Li and Lumb, 1987; Christian et al., 1994; Lacasse and Nadim, 1998; Griffiths and Fenton, 2004; El-Ramly et al., 2005; Cho, 2007; Suchomel and Masín, 2010; Wang et al., 2009; Reale et al., 2015).

The natural variability is associated to the randomness of the natural processes, evidenced as a variation over time and in space, which can be approximated through physical-mathematical models. There is also uncertainty due to the lack *of data and errors associated to their collection, both in the sampling methodology, data manipulation errors or transcription errors (Hidalgo and Pacheco, 2011), and the non-representativeness of the test regarding the natural conditions of the soil sample.*

Granite rocks, mainly coarse and pale color ones, are hard and dense when fresh, and practically waterproof. In Chile, these rocks cover wide extensions, both in the Coastal Mountain Range (Coastal Batholith) and the Andean Mountains. The Coastal Batholith is a big mass of intrusive rock, parallel to the subduction zone, derived from the cooling of the magma at kilometers of depth, which extends along the Coastal Mountain Range, from Valparaiso to the Nahuelbuta Mountain. Moreover, it shows preferential fracturing in the area adjacent to the coast, which allows a small underground water flow. And, together with anthropic actions such as road cuts, highways, quarry extractions, granodiorite and tonalite, it has been exposed to a strong process of physical and chemical weathering that has loosened and split the unions between quartz crystals, mica, feldspar, ferromagnesian minerals and orthoclase, which originates gravel with sandy texture, locally called Maicillo (Toro, 2007; Rodríguez, 2015).

2.2 Probability density function

As in all weathered granite rocks, the residual soil classification varies from coarse sand to clayey or silty sand, although rock fragments can be found inside the material. The weathering and alteration processes of the rocks are controlled by grain size and the mineralogy of the parent rock, micro fractures and diaclase formation, rain and infiltration, leaching, weathering history and duration (El-Ramly et al., 2005). The high randomness that the same material can show when weathered, produces a short-distance significant variability, even within uniform geological formations. Given this broad variability, the shear strength properties can be modeled as functions dependent from probability, which can be estimated from data points that are relatively close, considering the spatial location or rock origin, among others.

A goodness of fit analysis was made using the Kolmorov-Smirnov method, in order to determine the probability density function of the shear strength parameters, with a 5% confidence level. Therefore, 24 friction angle data and 24 cohesion data were used, which were obtained from direct shear tests in granite residual soils of the southern Coastal Batholith in Chile.

Data show that the effective friction angles of the granite soils vary between 23.5° and 38.5°. The average and the standard deviation are 30.6° and 3.6°, respectively. The value of the effective cohesion ranges between zero 4 and 24 kPa. The average and the standard deviation are 10.7 and 5.44 kPa, respectively. Figure 2 and Figure 3 show the probability histograms and the cumulative distribution function of the friction angle and the effective cohesion, respectively.

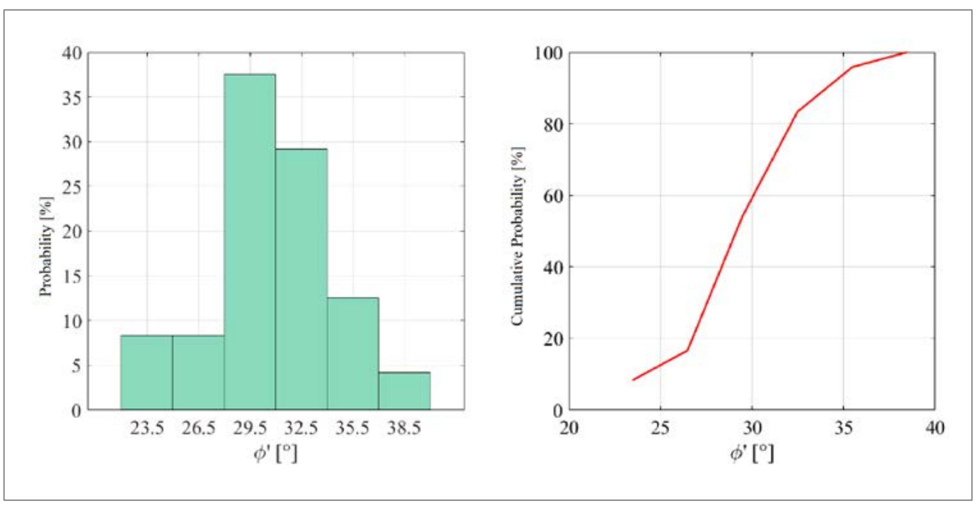

Figure 2. *Histogram and cumulative probability curve for the angle of internal friction*

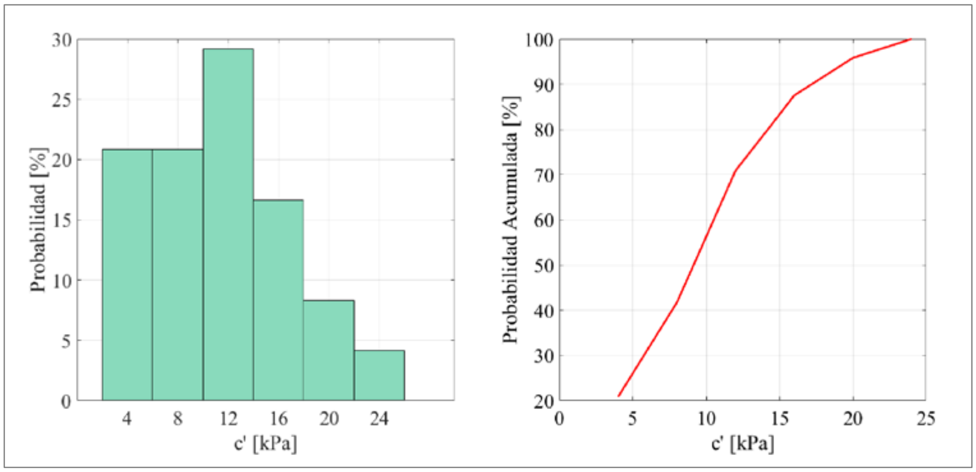

Figure 3. *Histogram and cumulative probability curve for the effective cohesion*

The probability distributions calculated for friction angle and cohesion, based on data of the site of Concepción, seem a reasonable representation of the variability of shear strength parameters typical from weathered granite residual soils. However, the average values and the variations of the shear strength parameters in a specific site may differ from the regional distributions. The variability within the site (in this case, the city of Concepción) tends to be lower than the variability of regional data (Zhang et al., 2005). When no specific information of the site is available, the uncertainty regarding the shear strength can be represented by regional distributions (El-Ramly et al., 2005).

3. Slope stability probabilistic analysis

In the past years, the safety concept and its assessment have undergone a remarkable evolution. Nevertheless, the determination of a global safety factor is still widely used in geotechnical structure designs (foundations, retaining walls, slopes, tunnels, etc.), which brings up additional difficulties to understand the influence on the design, because of the uncertainties concerning different geotechnical properties. Therefore, following the updating imposed by European and North American regulations, the deterministic methodologies are being replaced by more rational approaches, such as semi-probabilistic methods (for example, the partial coefficient method) and probabilistic methods based on the reliability theory (Pinheiro et al., 2014).

It should be noted that the slope stability probabilistic analysis offers the main advantage of logically considering the system's reliability and risk. Thus, the probabilistic models enable the development of new perspectives regarding risk and reliability that are beyond the scope of conventional deterministic models.

This study presents a procedure for probabilistically evaluating the slope stability in granite residual soils. The procedure is based on the Monte Carlo simulation to generate random data of shear strength parameters of the soil, considering their variability. Afterwards, a software for limit equilibrium analysis is run, once the probability distributions of the safety factor, the probability of failure and the reliability index for different slope geometries have been obtained.

3.1 Monte Carlo simulation

An alternative way to probabilistically evaluate the slope stability problem is the Monte Carlo simulation, where discrete values of random variables are generated consistently with their probability distribution, and the performance function is calculated for each set of generated data. The process is repeated many times in order to obtain a discrete and approximate probability density function of the performance function.

The Monte Carlo simulation allows obtaining a large number of random data once the probability distribution of the input variables are known, which in this case is the friction angle and the effective cohesion, as well as their averages and standard deviations. As data are generated, the slope stability is analyzed by deterministic methods, which also enables to determine the measures of central tendency corresponding to the safety factor, as well as the corresponding probability of failure (Cho, 2007; Hidalgo and Pacheco, 2011). The Monte Carlo simulation is a quantitative technique that uses statistics to simulate, through mathematical processes, the random behavior of real systems (Sandoval, 1987). The main advantage of this method is that it is relatively easy to implement with computers, besides handling a wide range of functions. The main disadvantage is that, according to the analyzed problem, it can converge slowly (Baecher and Christian, 2003).

The Monte Carlo simulation is used to generate random data of the soil's shear strength parameters, with the help of the MATLAB software for 100,000 iterations. Figures 4 and 5 show the probability histograms and the cumulative distribution functions for random data generated by the Monte Carlo simulation for friction angle and effective cohesion, respectively. A normal parametric distribution represented in the same chart closely agrees with the simulated experimental distribution function, both for the friction angle and the cohesion.

In order to generate random variables with any given distribution, first it is necessary to generate random numbers that are uniformly distributed between 0 and 1. The basis of *generating these numbers consists in generating whole, uniformly distributed random numbers. The generation of random numbers is made through the Park and Miller method (1988), corresponding to a multiplicative linear congruential method.*

3.2 Limit equilibrium method

Slope stability problems are usually analyzed with limit equilibrium methods of slices. The soil mass that fails is divided into several vertical sections to calculate the factor of safety, which is defined as the relationship between the available shear strength and the mobilized shear stress to maintain the static equilibrium. The static equilibrium of the slices and the soil mass as a whole are used to solve the problem (Cho, 2007). The limit equilibrium method used in this work corresponds to the method developed by (Morgenstern and Price, 1965), which was one of the first ones to satisfy all the equilibrium equations. The analysis shows the probabilistic evaluation of the critical slip surface of slopes in granite residual soils.

3.3 Case study

The slope stability probabilistic analysis considers that slopes are homogenously formed by residual soil. Figure 6 shows the slope geometry. The Monte Carlo simulation was carried out for slopes between 2m and 10m high, and an inclination between 45° and 85°.

The limit equilibrium analysis through the Morgenstern and Price method (1965) is made with the SLIDE 6.0 software, and afterwards, the results are processed with MATLAB. The shear strength parameters of the residual soil used for the limit equilibrium analysis are indicated in Figure 4 and Figure 5.

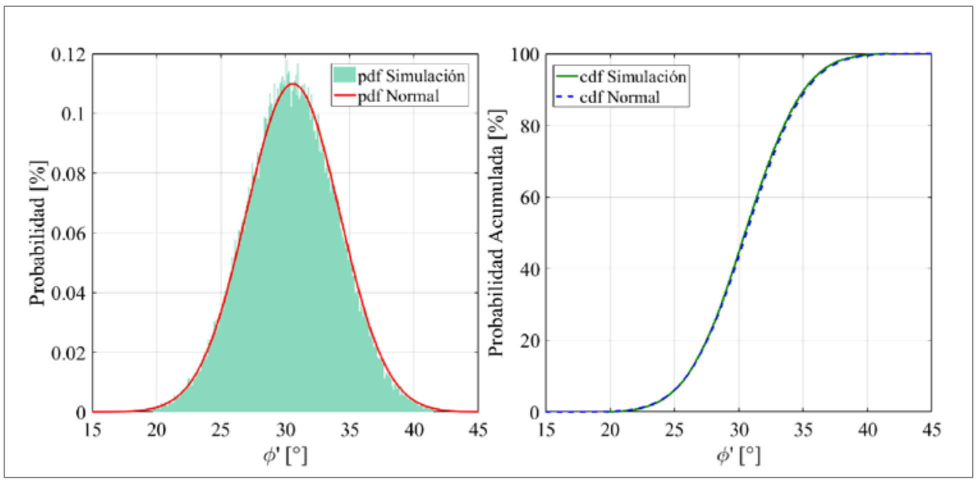

Figure 4. *Histogram and cumulative probability curve of data generated by the Monte Carlo simulation for the angle of internal friction*

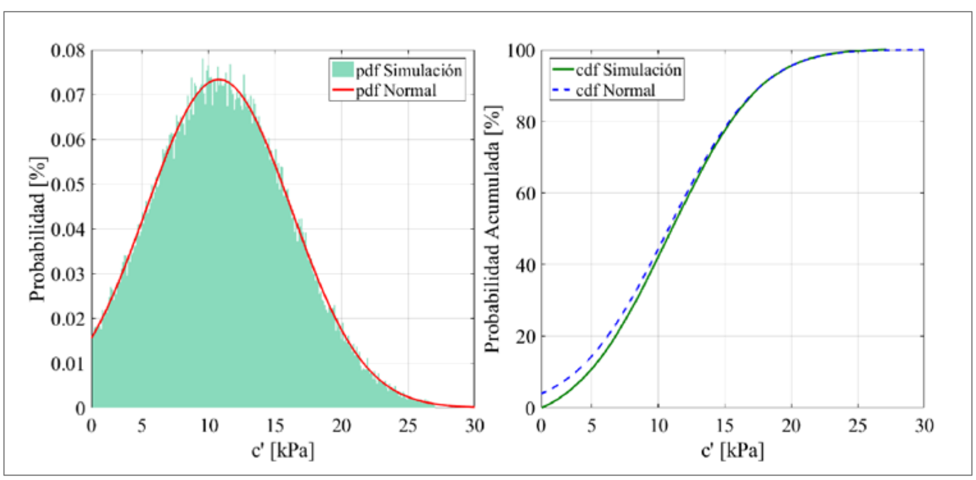

Figure 5. *Histogram and cumulative probability curve of data generated by the Monte Carlo simulation for the effective cohesion*

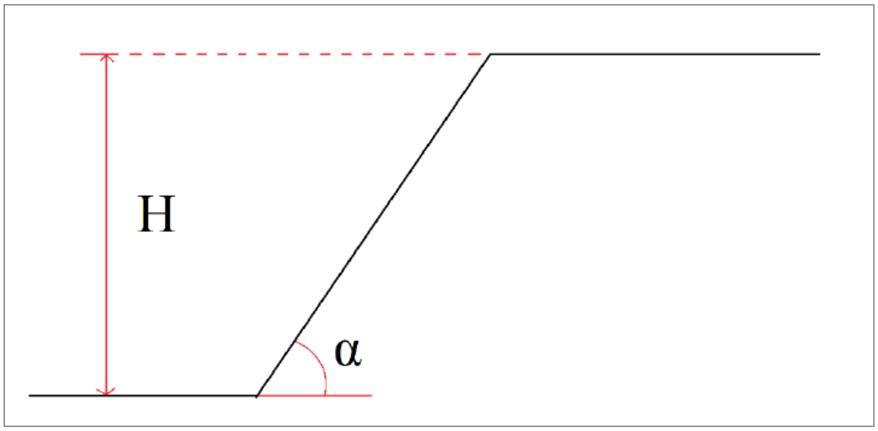

Figure 6. *Slope geometry, H and α correspond to the slope height and inclination, respectively*

4. Results

The results of the slope stability probabilistic analysis in granite residual soil are presented in terms of the probability of failure, the factor of safety and the reliability index.

4.1. Probability of Failure

Table 1 shows the values obtained from the probability of failure for each analyzed case, while Figure 7 shows the probability of failure in a logarithmic scale based on the slope's inclination angle. It is observed that, for all analyzed heights, and slope inclination equal to 45°, the probability of failure is lower than 5%, while for heights over 5m and greater inclinations, the probability of failure exceeds 80%, which shows that they are clearly unstable. It is also possible to observe that the most stable critical height is 2m, with a probability of failure lower than 2% for all slope inclinations.

Table 1. *Probability of failure (%)*

	2 m	3 m	4 m	5 m	6 m	7 m	8 m	9 m	10 m
45°	0	0.007	0.078	0.244	0.543	0.977	1.732	2.312	3.87
50°	0.01	0.105	0.368	1.146	2.612	3.929	4.822	7.334	10.332
55°	0.007	0.193	2.892	3.234	6.804	9.385	13.579	17.776	22.655
60°	0.01	0.274	5.455	11.454	19.505	22.296	31.161	39.459	42.149
65°	0.01	0.57	3.668	11.94	20.843	46.776	57.676	71.431	72.862
70°	0.435	0.792	5.001	16.952	34.626	54.838	71.343	84.786	92.142
75°	1.227	7.363	22.793	39.257	47.184	64.351	75.509	96.721	98.928
80°	0.769	9.605	29.787	65.159	87.279	96.416	98.597	99.456	99.861
85°	0.469	21.842	34.61	70.853	89.368	99.512	99.956	99.912	99.961

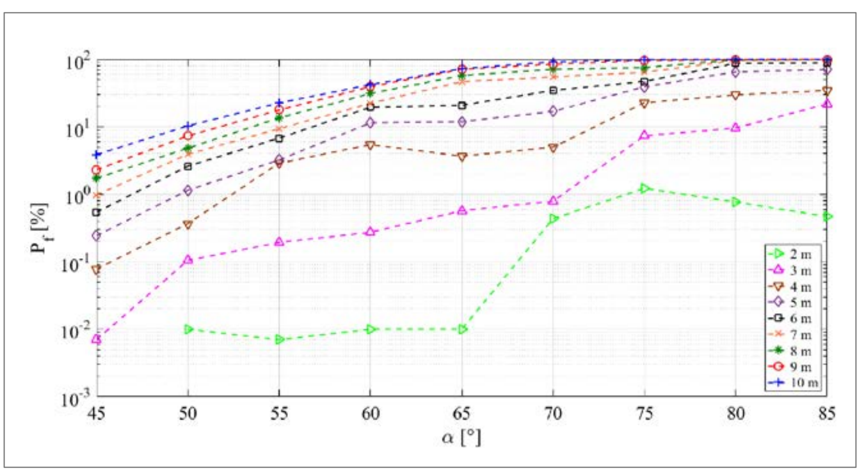

Figure 7. *Probability of failure for different slope inclinations and heights*

4.2 Factor of safety

Table 2 and Table 3 indicate the safety factor values for the probabilistic and deterministic cases, respectively. Figure 8 shows the relationship between both safety factors.

Figure 8 shows an apparently linear relationship between the deterministic factor of safety and the average of the probabilistic factor of safety, where the former presents higher values than those obtained with the Monte Carlo simulation. Results show that as the probability decreases, both factors of safety tend to resemble among them. Instead, when the probability increases, not only the deterministic values are higher, but the dispersion between each case also increases. Despite this, the results obtained by probabilistic methods are much more conservative than the results obtained by the average of strength parameters.

Table 2. *Average value of the factor of safety*

	2 m	3 m	4 m	5 m	6 m	7 m	8 m	9 m	10 m
45°	3.19	2.593	2.2	1.952	1.779	1.672	1.555	1.496	1.415
50°	2.437	2.19	1.972	1.783	1.51	1.463	1.424	1.35	1.29
55°	2.441	1.857	1.57	1.522	1.416	1.351	1.28	1.225	1.177
60°	2.297	1.804	1.448	1.296	1.181	1.172	1.106	1.053	1.042
65°	2.211	1.679	1.415	1.247	1.152	1.015	0.971	0.915	0.906
70°	2.025	1.615	1.365	1.185	1.07	0.985	0.922	0.865	0.825
75°	1.818	1.464	1.193	1.062	1.015	0.93	0.88	0.787	0.748
80°	1.758	1.337	1.11	0.939	0.837	0.768	0.73	0.702	0.669
85°	1.722	1.229	1.073	0.925	0.844	0.716	0.653	0.596	0.55

Table 3. *Factor of safety*

	2 m	3 m	4 m	5 m	6 m	7 m	8 m	9 m	10 m
45°	3.304	2.521	2.141	1.901	1.762	1.634	1.521	1.459	1.383
50°	3.058	2.35	1.986	1.751	1.624	1.506	1.391	1.317	1.261
55°	2.941	2.239	1.844	1.627	1.474	1.365	1.277	1.212	1.156
60°	2.785	2.107	1.746	1.514	1.372	1.262	1.177	1.113	1.059
65°	2.673	1.976	1.642	1.431	1.3	1.174	1.095	1.025	0.975
70°	2.558	1.879	1.548	1.345	1.204	1.096	1.02	0.954	0.911
75°	2.271	1.681	1.386	1.214	1.092	0.999	0.936	0.876	0.833
80°	2.128	1.551	1.274	1.096	0.97	0.893	0.829	0.777	0.738
85°	2.039	1.489	1.208	1.038	0.927	0.844	0.764	0.696	0.653

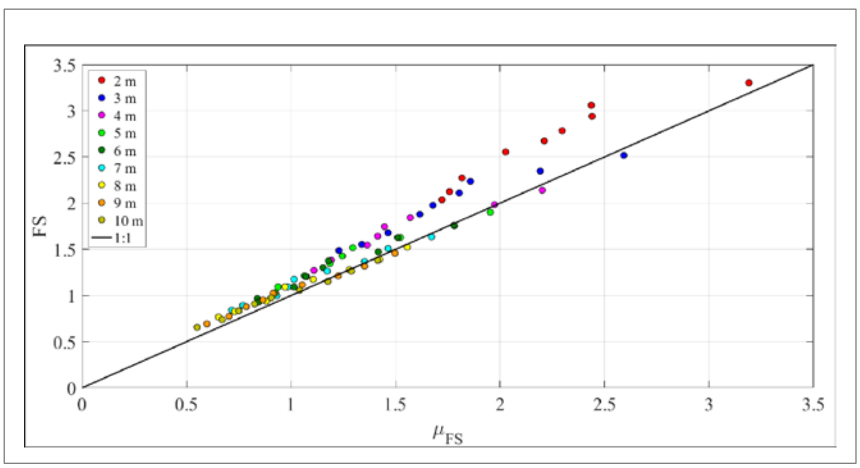

Figure 8. *Average of the factor of safety (probabilistic case) against the deterministic factor of safety*

4.3 Reliability index

A complementary probabilistic analysis consists in obtaining the reliability index, which is defined as the capacity of a system to do the required functions, under conditions established for a specific period of time (Wang and Constantino, 2009). In other words, this index describes the safety of the system, according to the number of standard deviations that separate the best estimation of the said variable from its failure value (Christian et al., 1994), that is, a factor of safety equal to 1.0. The advantage of the reliability index is that it does not require knowing the distribution of the analyzed parameter. However, there is a tacit assumption regarding its distribution, because it can be approximated through the average values and the standard deviation.

One of the main problems when doing a reliability analysis is to define acceptable safety levels. In this perspective, the problem is to define the maximum probability of failure that a structure can tolerate. Currently, there is no consensus regarding this matter, and USACE (1999) has presented the most known proposal, which defines the limits of the reliability index, and therefore, the maximum probability of failure of a structure (see Table 4). Likewise, Dell'Avanzi and Sayão (1998) define the acceptable probability of failure in different types of geotechnical structures (see Table 5).

Table 4. *Reliability index and probability of failure (USACE, 1999)*

Expected Performance	Reliability index	Probability of failure (%)
High	5	3×10^{-5}
Good	4	3×10^{-3}
Above the average	3	10^{-1}
Bellow the average	2.5	6×10^{-1}
Poor	2	2.3
Unsatisfactory	1.5	7
Dangerous	1	16

Table 5. *Reliability index and probability of failure (Dell'Avanzi and Sayão, 1998)*

Case	Reliability index	Probability of failure (%)
Foundations	2.3 – 3.0	$1 - 10^{-1}$
Mining slopes	1.0 – 2.3	$10 - 10^{-1}$
Dams	3.5 – 5.0	$10^{-1} - 10^{-3}$
Retaining structures	2.0 – 3.0	$10^{-1} - 10^{-1}$

Table 6 indicates the reliability index values calculated for each analyzed case, while Figure 9 shows these results graphically. As observed, the reliability index is theoretically related, by a standard normal distribution, with the probability of failure (Christian et al., 1994), which is consistent with the experimental results. Figure 9 shows that, for reliability index values lower than 1.5, there is good fitness with the results obtained for each slope configuration, which coincides with the observations made by Christian et al (1994), who mention that, with reliability index values higher than 1.5 to 2.0, the probability of failure differs substantially from the theoretical value. Additionally, with a reliability index value equal to 1.5, the trend of the reliability index not only stops following a standard normal distribution, but also the dispersion increases as the probability of failure turns smaller, even though the reliability index does not show a great variation. This can cause problems when designing according to some criteria, because, as observed in the case of a 3m-high slope, the reliability index does not vary for probabilities of failure between 0.1% and 0.04%.

Table 6. *Reliability index*

	2 m	3 m	4 m	5 m	6 m	7 m	8 m	9 m	10 m
45°	2.863	2.381	2.280	2.199	2.149	2.008	1.880	1.811	1.654
50°	2.568	2.403	2.203	1.964	1.864	1.720	1.573	1.412	1.251
55°	2.765	2.421	1.851	1.805	1.525	1.351	1.144	0.973	0.805
60°	2.736	2.368	1.637	1.270	0.909	0.815	0.531	0.298	0.220
65°	2.751	2.251	1.745	1.212	0.847	0.091	-0.188	-0.572	-0.610
70°	2.196	2.185	1.624	0.993	0.428	-0.098	-0.560	-1.043	-1.444
75°	2.057	1.514	0.811	0.297	0.078	-0.388	-0.716	-1.887	-2.355
80°	2.163	1.364	0.571	-0.384	-1.155	-1.804	-2.186	-2.503	-2.884
85°	2.252	0.864	0.435	-0.536	-1.262	-2.710	-3.467	-4.290	-5.142

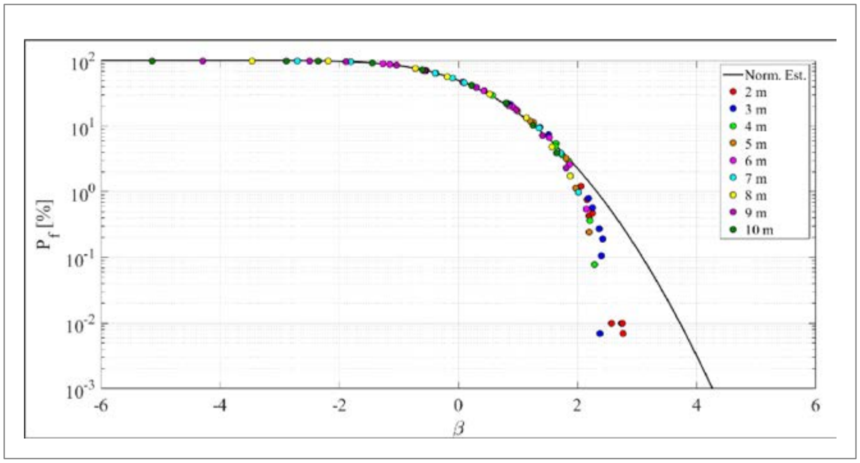

Figure 9. *Relationship between the reliability index and the probability of failure*

5. Conclusions

The results obtained through the probabilistic analysis confirm the relationship between slope height and inclination and the general instability of the slope. The most inclined is the slope, the bigger is the instability and, at the same time, the probability of failure is higher. The same analogy can be applied in relation to the slope height.

In Chile, the value of the minimum safety factor used for slope design ranges between 1.40 and 1.50. Among the studied cases, and regarding the configurations that complied with this requirement, it is observed that their probability of failure is lower than 3%, thus fulfilling the acceptable probability of failure proposed by Dell'Avanzi and Sayão (1998).

The curve of probability and reliability index shows that the relationship between both parameters perfectly follows a standard normal distribution up to a reliability index close to2.0. Therefore, for study cases where the maximum probability of failure is lower than 5%, it should be taken into account that the variation of the reliability index will be a small one in relation to low values of probability of failure.

The probabilistic analysis is a useful tool that can be currently applied thanks to the technological progress, and it offers the advantage of including the uncertainty of nature in deterministic problems. The use of both methodologies will be beneficial for the geological and geotechnical engineering, not only because it allows better choosing the safety factor, but also because they widen the vision to address these problems, thus increasing the number of factors involved in the analysis and improving the decision-making process. However, it is necessary to be careful with the simplifications used in the probabilistic analysis, which can lead to underestimate the probability of failure of the slope. The new challenge in this matter lies in the acceptable risk determination, that is, in defining the acceptable probability of failure to evaluate the stability of a slope or structure and what criteria should be used to define it.

6. References

Alonso E. (1976), *Risk analysis of slopes and its application to slopes in Canadian sensitive clays. Geotechnique, 26: 453–472.*

Baecher G., Christian J. (2003),, *Reliability and Statistics in Geotechnical Engineering. John Wiley and Sons Ltd. England. 605 p.*

Bhattacharya G., Jana D., Ojha S., Chakraborty S. (2003), *Direct search for minimum reliability index of earth slopes. Comp and Geotech Journal 30:455–462.*

Brady N., Weil R. (1996), *The nature and properties of soils. Upper Saddle River. Prentice Hall, NJ.*

Cabrera T. (2007), *Características Geotécnicas de los suelos residuales del batolito de la costa de la cordillera de la Costa. Tesis de Ingeniería Civil, Universidad de Chile.*

Cho S. (2007), *Effects of spatial variability of soil properties on slope stability. Engineering Geology Journal 92: 97–109.*

Christian J., Ladd C., Baecher G. (1994), *Reliability applied to slope stability analysis. Journal of Geotechnical Engineering 120 (12): 2180–2207.*

Dell'Avanzi E., Sayão A. (1998), *Avaliação da probailidade de rupture de taludes. Congresso Brasileiro de Mecânica dos Solos e Engenharia Geotécnica, 11, Vol. 2: 1289–1295.*

Elkateb T., Chalaturnyk R., Robertson P. (2003), *An overview of soil heterogeneity: quantification and implications on geotechnical field problems. Canadian Geotechnical Journal 40 (1): 1–15.*

El-Ramly H., Morgenstern N., Cruden D. (2005), *Probabilistic assessment of stability of a cut slope in residual soil. Geotechnique 55, No. 1: 77–84.*

Flandes N. (2017), *Estudio de la relación entre meteorización y características geo-mecánicas de la roca granítica de Concepción. Tesis de Ingeniería Civil Geológica y Magister en Ingeniería Geotécnica, Departamento de Ingeniería Civil Universidad Católica de la Santísima Concepción.*

Griffiths D., Fenton G. (2004), *Probabilistic slope stability analysis by finite elements. Journal of Geotechnical and Geoenvironmental Engineering 130 (5): 507–518.*

Hidalgo C., Pacheco A. (2011), *Herramientas para el análisis por confiabilidad en geotecnia: Aplicación. Revista Ingenierías, Universidad de Medellín 10, 8: 79–86.*
Lacasse S., Nadim F. (1998), *Risk and Reliability in Geotechnical Engineering. 4th International Conference on Case Histories in Geotechnical Engineering. Paper No. SOA-5: 1172 –1192.*
Li K., Lumb P. (1987), *Probabilistic design ofslopes. Canadian Geotechnical Journal 24: 520–531.*
Morgenstern N., Price V. (1965), *The Analysis of the stability of general slip surfaces. Geotechnique 15, 1: 79–93.*
Park S., Miller K. (1988), *Random number generators: good ones are hard to find. Communications of the ACM 31 (10): 1192–1201.*
Pinheiro R., Ramos L., Teixeira J., Alfonso M., Chaminé H. (2014), *MGC – RocDesign|CALC: a geomechanical calculator tool for rock design. In: Alejano, L., Perucho, A., Olalla, C., Jiménez, R. (eds) Proceedings of Eurock2014, Rock Engineering and Rock Mechanics: Structures in and on Rock Masses (ISRM European Regional Symposium, Vigo, Spain). CRC Press/Balkema, Taylor & Francis Group, London. Pp 655 – 660.*
Poblete M., Dobry R. (1968), *Modelo dinámico del suelo de Concepción. Revista IDIEM 7, 6: 12-18.*
Reale C., Xue J., Pan Z., Gavin K. (2015), *Deterministic and probabilistic multi-modal analysis of slope stability. Comp and Geotech Journal 66: 172–179.*
Rodríguez P. (2015), *Caracterización geomecánica y mineralógica del maicillo en la Cordillera de Nahuelbuta. Tesis de Ingeniería Civil, Departamento de Ingeniería Civil, Universidad Católica de la Santísima Concepción.*
Ruíz C. (2002), *Determinación de los parámetros CIU en muestras inalteradas de Maicillos graníticos residuales de Concepción. Tesis de Ingeniería Civil, Universidad de Concepción.*
Sandoval M. (1987), *Dinámica de los procesos hidrológicos en la Cuenca del rio Ñuble en San Fabian. Tesis para optar al título de Ingeniero Civil. Universidad Católica de la Santísima Concepción, Facultad de Ingeniería, Departamento de Ingeniería Civil.*
Suchomel R., Masín D. (2010), *Comparison of different probabilistic methods for predicting stability of a slope in spatially variable c–φ soil. Comp and Geotech Journal 37: 132–140.*
Tang W., Yucemen M., Ang A. (1976), *Probability-based short-term design of soil slope. Can Geotech Journal 13:201-215.*
Toro, K. (2007) *Influencia de las características geológicas en las propiedades geotécnicas de graminoides jurásicos y suelos asociados en la Ruta 68. Tesis de Geología, Universidad de Chile.*
USACE (1999), *Risk – based analysis in geotechnical engineering for support of planning studies. ETL 1110–2–556.*
Villalobos S. (2011), *Análisis y diseño de una excavación apernada en un suelo residual de Concepción. Tesis de Ingeniería Civil, Departamento de Ingeniería Civil, Universidad Católica de la Santísima Concepción.*
Villalobos S., Oróstegui P., Villalobos F. (2013), *Re-assessing a soil nailing design in heavily weathered granite after a strong earthquake. Bull Eng Geol Environ 72:203–212, DOI 10.1007/s10064-013-0466-7*
Wang W., Constantino C. (2009), *Reliability analysis of slope stability at nuclear power plant site. 20th International Conference on Structural Mechanics in Reactor Technology. SMiRT 20 – Division 7, Paper 1982: 1–10.*
Wang Y., Cao Z., Au S. (2010), *Efficient Monte Carlo Simulation of parameter sensitivity in probabilistic slope stability analysis. Comp and Geotech Journal 37: 1015–1022.*
Zhai Q., Rahardjo H., Satyanaga A. (2016), *Variability in unsaturated hydraulic properties of residual soil in Singapore. Engineering Geology Journal 209: 21–29.*
Zhang L., Zhang L., Tang W. (2005), *Rainfall-induced slope failure considering variability of soil properties. Geotechnique 55, No. 2: 183–188.*