

Optimización metaheurística de conjuntos estructurales de hormigón armado

Metaheuristic optimization of structural sets of reinforced concrete

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Abstract

This paper presents the economic structural optimization of the Casa Síndico project using an algorithm programmed through the CSI API functions SAP2000v19-MATLAB R2015a, applying metaheuristic techniques: Genetic Algorithms (GA) and Particle Swarm Optimization (PSO), in addition to hybridization between them. The results show that PSO has a better performance than GA for this type of optimization, although both, working with their simple methodologies, are not completely efficient, which is verified when creating and applying a hybridization between the two, using GA to create an initial swarm for PSO to carry out the optimization process, obtaining results of up to 10% better. Regarding the structural results, a direct cost of construction is obtained by 13% more economical when applying the proposed methodology, leaving, for the beams, heights of relation L/h between 15 and 17.5, for the columns, the use of sections with rectangularities of up to 1.35, in the direction that more flexion occurs, something similar to what happens for the foundations, where the rectangularity of these follows the previous criterion, obtaining values of up to 1.4.

Keywords: Structural optimization, structural set, Metaheuristics, Genetic Algorithms, Particle Swarm Optimization

Resumen

En este artículo se presenta la optimización estructural económica del proyecto Casa Síndico utilizando un algoritmo programado mediante las funciones CSI API SAP2000v19-MATLAB R2015a, aplicando técnicas metaheurísticas: Algoritmos Genéticos (GA) y Optimización por Enjambre de Partículas (PSO), además de una hibridación entre estas. Los resultados muestran que PSO presenta un mejor comportamiento que GA para este tipo de optimización, aunque ambos, trabajando con sus metodologías simples, no resultan del todo eficiente, lo cual se comprueba al crear y aplicar una hibridación entre los dos, utilizando GA para crear un enjambre inicial para que PSO realice el proceso de optimización, obteniéndose resultados hasta un 10 % mejores. En cuanto a los resultados estructurales, se obtiene un costo directo de construcción un 13 % más económico al aplicar la metodología propuesta, quedando, para las vigas, peraltos de relación L/h entre 15 y 17.5, para las columnas, el uso de secciones con rectangularidades de hasta 1.35, en la dirección que ocurre más flexión, algo parecido a lo que ocurre para los cimientos, donde la rectangularidad de estos sigue el criterio anterior, obteniéndose valores de hasta 1.4.

Palabras clave: Optimización estructural, Conjunto estructural, Metaheurísticas, Algoritmos Genéticos, Optimización por Enjambre de Partículas

1. Introduction

The development of researches in the field of structures has significantly contributed to the achievement of increasingly rational projects, which always aim at safety-cost relationships closer to the real optimum.

In recent years, and thanks to the development of interactive or automated computer techniques, the civil engineering branch known as “structural optimization” has been given a boost, which has allowed improving the designs, thereby reducing costs, materials and time in these processes (Negrin, 2019).

Furthermore, it is essential to understand the structural optimization as the result of optimizing the whole set, since it has been demonstrated that the optimization of individual elements omits a key aspect of the concept of structure, which is the influence of each individual element on the whole, where load distributions are strongly influenced by this aspect (Negrin, 2014) (Negrin, 2016) (Negrin, 2019).

Many authors have done research about methodologies for optimizing structures, either single or multi-objective, but these procedures have still several constraints.

Most of the methodologies proposed by different authors ignore the soil-structure interaction (SII), as well as the optimization of the foundations within the structural set, (Paya, 2007) (Borda and Rodríguez, 2010) (Kripka et al., 2013) (Kulkarni and Bhusare, 2016) (Mejía, and Orozco, 2019), despite the fact that it has been demonstrated that the SII modify the load distribution of the superstructure in a considerable proportion (Chagoyén et al., 2010); and it should also be considered that foundations represent a great deal of the cost of the structure (Negrin M. 2014) (Negrin, 2019). On the other hand, different authors are focused on the optimization of isolated elements (Borda and Rodríguez, 2010) (Kulkarni and Bhusare, 2016), and do not take into account the interaction among them. A further limitation is the optimization of flat arcades (Paya, 2007) (Borda and Rodríguez, 2010) (Mejía and Orozco, 2019), ignoring the great load requirement differences between these structures and most of the real spatial structures. Some works suggest that it is more rational to establish more geometry sets (different types of sections) and design sets (Kripka et al., 2013), although they leave out the ease-of-construction criterion. And other methodologies are concentrated in

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obtaining the minimum structure weight, where the process starts with small sections and progressively increases the dimensions, until the strength and stiffness criteria are met, although poor results are obtained, such as beams with square sections (Mejía and Orozco, 2019).

2. Materials and Methods

2.1 Mathematical Formulation of the Structural Optimization Problem

The mathematical formulation of the optimization problem is a crucial step in the process, since it defines the objective function(s), variables, constraints and parameters assigned.

2.1.1 Definition of the Objective Function

The objective function obviously depends on the optimization criterion you decide to use. The chosen criterion was minimum total cost, which consequently defined the total direct cost of the structure (Negrin, 2014), based on the following items (PRECONS.II 2008):

$$C_{total} = C_{beams} + C_{col} + C_{cim} \quad (1)$$

Where:

C_{total} : Total cost (\$)

C_{beams} : Sum of beam costs (\$)

C_{col} : Sum of column costs (\$)

C_{found} : Sum of the foundation costs (\$)

The calculation of each direct building cost of the elements includes the formwork, manufacture and placement of longitudinal and transverse reinforcement, concrete manufacture and placement, and excavation and filling in the case of foundations.

2.1.2 Definition of the Variables

This research uses discrete variables, because, in order to make a more practical structural optimization, we need feasible final construction solutions (dimensions being multiple of 5, in cm), in addition to considerably reduce the solution space. For the study case of this research, 20 variables were defined, which are associated to the dimensions of cross-sections (beams and columns), rectangularity for the foundation and concrete compressive strength ($f'c$) for the different elements.

2.1.3 Identification of the Constraints

An essential part of the mathematical formulation of the whole optimization process are the constraints imposed to the problem, consistently with the minimum cost requirement. It is known that design variables depend on constraints, which limit their free circulation in the optimization process (Negrin, 2014). Constraints are divided into two groups: implicit and explicit, which play a different role within the optimization process.

Implicit constraints are input in the solution algorithm and are mainly associated to the fulfilment of the conditions imposed by the design (equations of state) through limit states (strength, stiffness). Explicit constraints, on the other hand, are mainly constructive and they limit the movement interval of the variables in the optimization process, a mandatory requirement of the optimization methods used in this research.

Everything described above is graphically illustrated in (Figure 1).

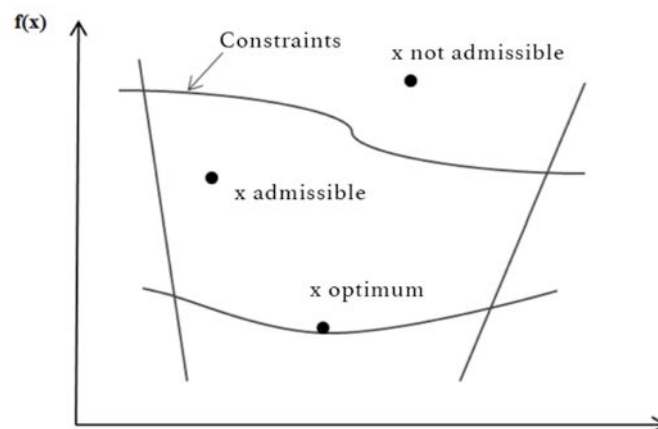


Figure 1. Graph of the optimization problem

2.1.4 Generated Response Surface

Due to its special characteristics, the structural optimization generates highly complex objective functions, which include many local optima. The (Figure 2) shows the evolution of these response surfaces through this kind of research in the Faculty of Construction of the Universidad Central Marta Abreu de las Villas in Cuba.

It can be appreciated that, as the complexity in the formulation of the optimization problem increases (higher

number of variables, use of discrete variables, inclusion of aspects such as how to take from the calculation area to real life, etc.), much more irregular surfaces are generated, thereby making it impossible to use classic optimization methods. Therefore, it is necessary to consider metaheuristic methods and even a combination thereof (Hybridization), with the purpose of obtaining satisfactory results in this searching process for optimum results.

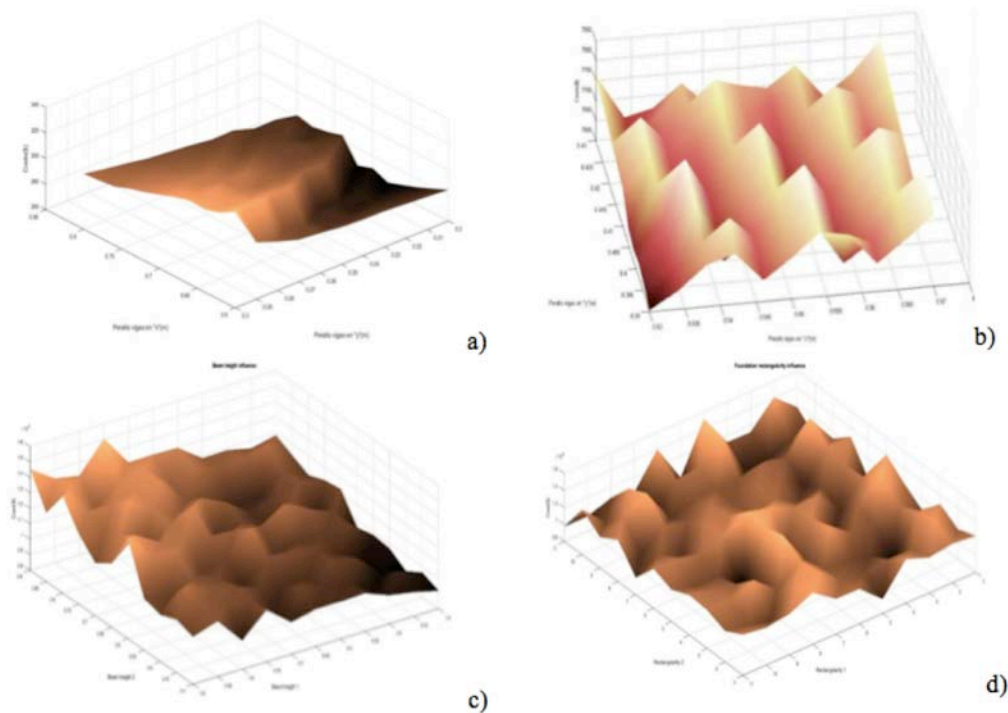


Figure 2. Evolution of the response surface generated in the structural optimization: a) Surface generated in (Negrin, 2016), b) surface generated in (Medina, 2017), c) surface generated in (Negrin, 2019) due to the influence of the beams' height, d) surface generated in (Negrin, 2019) due to the influence of the rectangularity of the foundations.

2.2 Heuristic and Metaheuristic Methods: GA and PSO

A metaheuristic method does not use a common and rigorous methodology to obtain a result. Generally speaking, these methods give an acceptable result in a reasonable time, and they can be also be applied when there is an objective function with multiple local optima or when these functions are composed by continuous and discrete intervals (Cujía, 2010).

2.2.1 Genetic Algorithms

Genetic algorithms are a tool for solving optimization problems, based on a natural selection that imitates the

biological evolution process. GA systematically modify a population of individuals; in each stage, the algorithm selects certain individuals to become "parents" and generate "children" in the following generation. Through successive generations, the population evolves towards an optimum solution (Figure 3). Their main advantage is that they can be used on a wide range of problems where classical optimization methods cannot deal with them properly; for example, problems with discontinuous, non-differentiable, stochastic or highly non-linear objective functions (Paya, 2007).

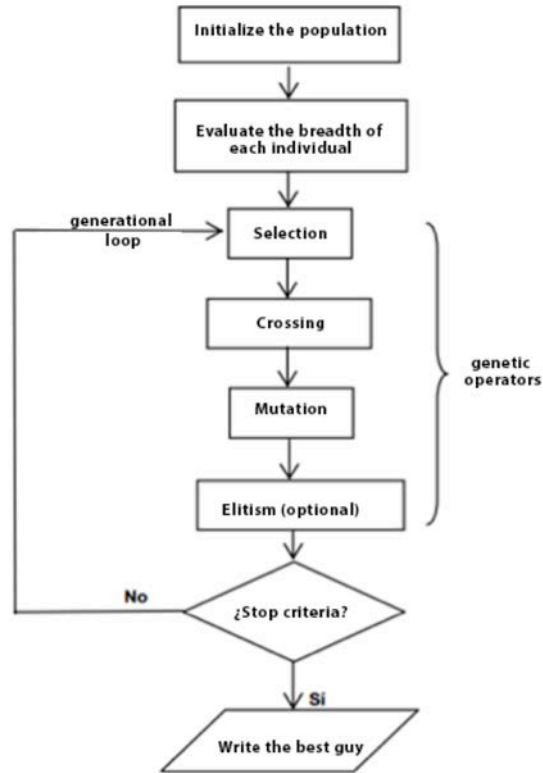


Figure 3. Diagram of a simple GA (Paya, 2007)

2.2.2 Particle Swarm Optimization

The Particle Swarm Optimization is a metaheuristic technique inspired by the social behavior of the flight of bird flocks or the movement of fish schools. It is based on factors influencing the decision making by an agent that is part of a

group of similar agents. Each agent makes the decision according to a social component and an individual component, which determines the movement (direction) of this agent to reach a new position in the solution space (Figure 4) (García, 2006).

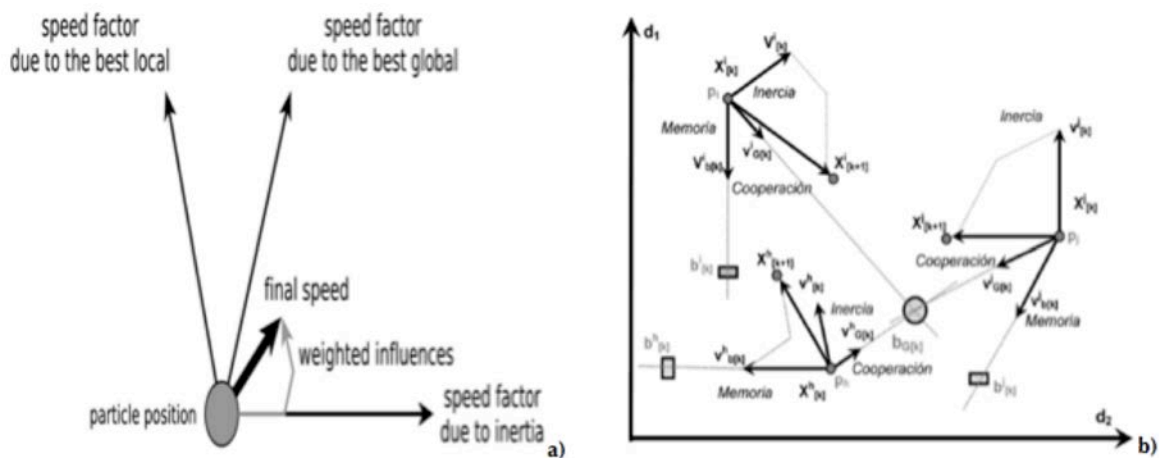


Figure 4. Graphical representation of the global PSO methodology, a) Movement of particles, b) Interaction between particles (Sancho, 2016)

This method works with one population (called cloud or swarm) of candidate solutions (called particles). This particles move around the search space following a couple of simple mathematical rules. The movement of each particle depends on their best position obtained, as well as on the

best global position found in the entire search space. Moreover, it is a multi-agent space, that is, particles are simple agents moving through the search space, who save and communicate the best solution found (Figure 4) (García, 2006).

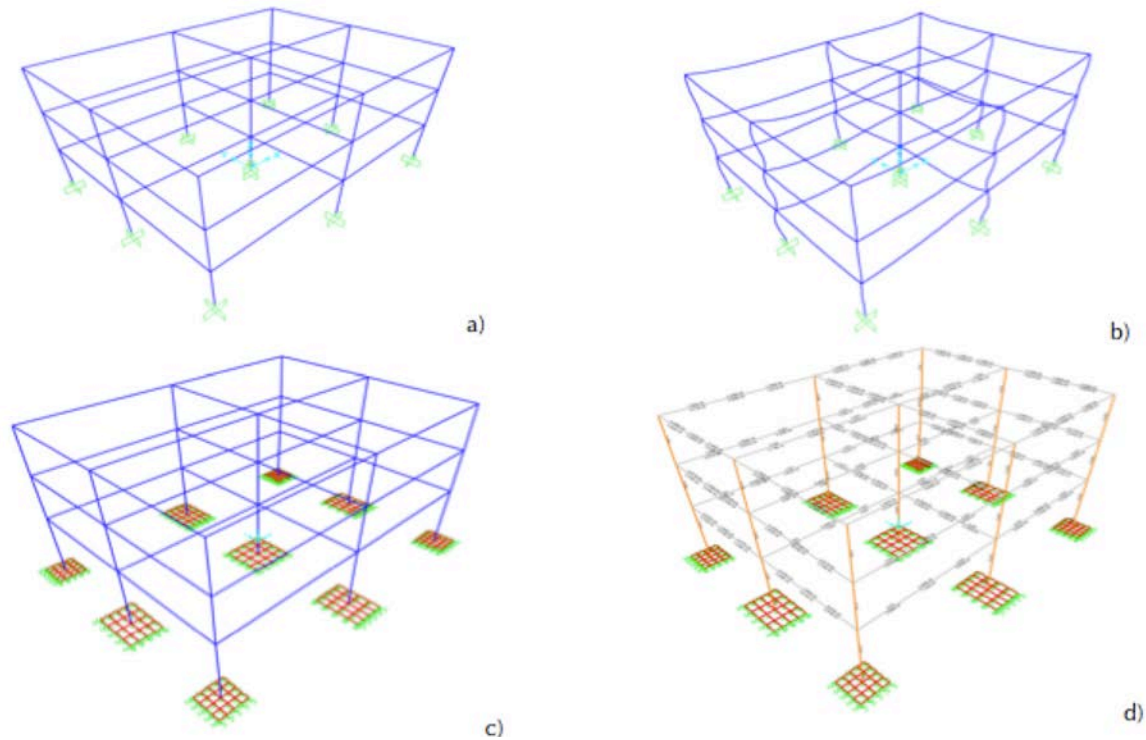


Figure 5. Main steps of each function count: a) Step 1, b) Step 2, c) Step 3 and d) Step 4

2.3 Algorithm for Solving the Optimization Problem through CSi API

The proposed methodology is completely automated, since it is programmed according to CSi API functions of the SAP2000-MATLAB software, which allow linking a programming language (MATLAB R2015a) with a software for structural modeling, analysis and design (SAP2000v19). In general, each function cycle or count (evaluate a specific series of variable values) can be divided into four essential

steps, see (Figure 5), formed and complemented by other sub-steps in them, see flowchart in (Figure 6), or from outside, as in the case of F_{Prom1} and F_{Prom2} , which are prior to the optimization process, follow the same steps in (Figure 5), and are used to coordinate the model with the algorithm and obtain the average cost of the structure, which in turn is used to penalize the objective function in case of not complying with some implicit constraints.

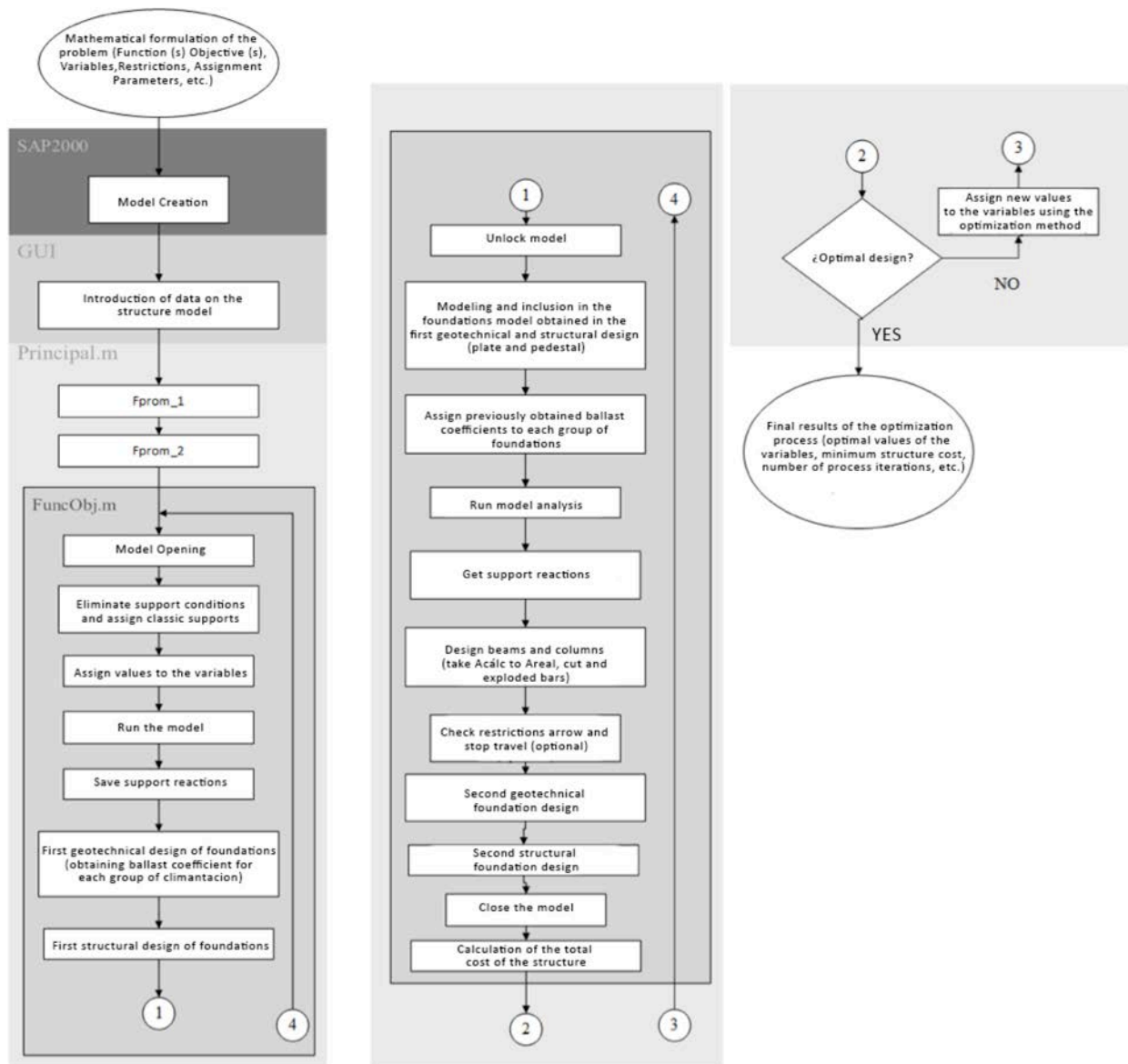


Figure 6. Global flowchart for the solution algorithm to the optimum design problem of structural sets using CSi API

The basic four steps of each function count are:

Step 1: Open the model, input the classic supports (embedment or articulation) and assign the new values of the variables (cross-section dimensions, etc.).

Step 2: Execute the analysis and save the support reactions in order to do the subsequent first geotechnical and structural design of the foundations (Quevedo 2000), in addition to the calculation of the ballast coefficient (k) for each foundation group (Chagoyén Méndex et al. 2018).

Step 3: Eliminate the classical supports and input the previously modeled foundations (footing and pedestal), as

well as the k coefficient for modeling the SII (Chagoyén Méndex et al. 2018).

Step 4: Execute the analysis again and design all the elements (beams, columns and foundations), in order to calculate the direct building costs of the whole structure.

This procedure is repeated as many times as the optimization algorithm requires to find the best solution. Thus, this algorithm rules the process, assigning new values to the variables depending on their operation and obtained results. It should be kept in mind that the proposed methodology includes the SII and the optimization of foundation designs, as explained above.

3. Results and discussion

3.1 Optimization of the Casa Síndico Project

The Casa Síndico Project is a real structure with a high structural complexity, composed by structural elements of different typology. Additionally, the floor plans have an

asymmetrical distribution, which introduces special characteristics to the elements' response to the load requirements, see (Figure 7). Overall, 116 beams, 99 columns and 23 foundations were optimized.

The results are divided into two basic groups: performance of optimization methods and structural

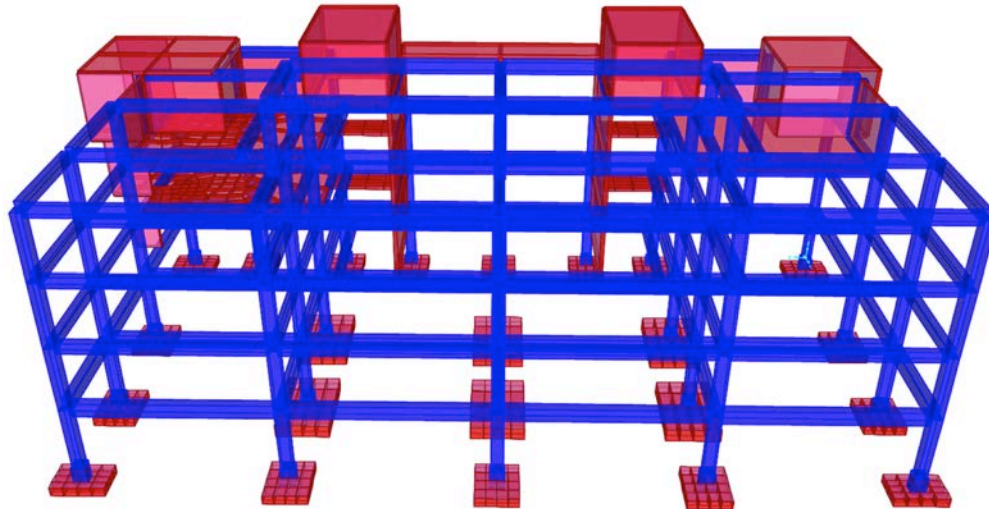


Figure 7. Model of the Casa Síndico Project

3.1.1 Results Associated to the Performance of Optimization Methods

This section compares the results when applying simple methods to a fictitious model, similar to that of (Figure 5), more simple than that of the main study, with the aim of

finding the best version to face the optimization of a much more complex model. In these optimization tests, four types of soil are used in order to give diversity to the study, using simple GA and PSO methodologies.

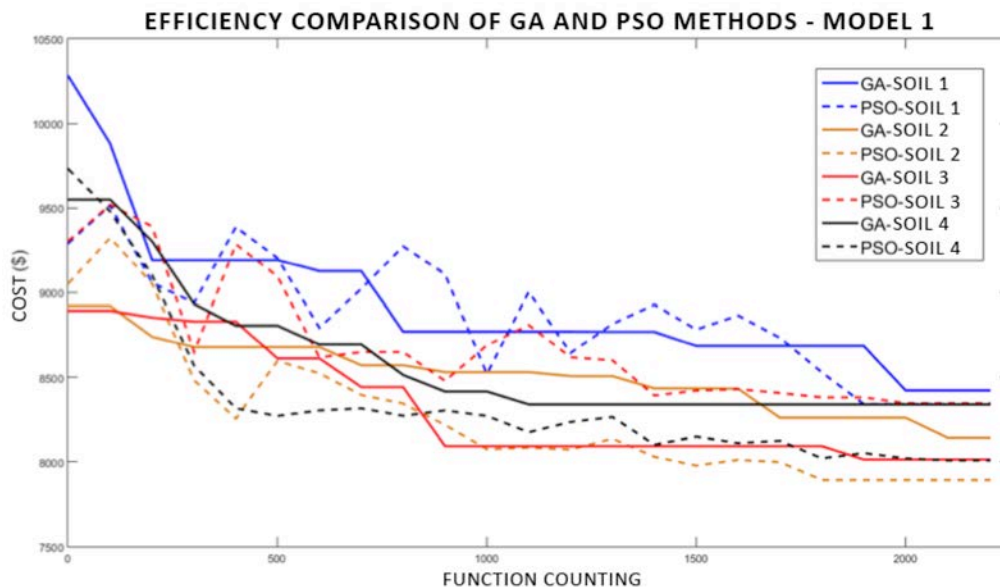


Figure 8. Comparison between simple GA and PSO methodologies in the optimization of a fictitious model for four types of soil



In (Figure 8) we can appreciate the superiority of PSO over GA, while obtaining better results in three of the four tests carried for each one. However, the results were not stable, which led us to wonder whether these methods, with their simple methodologies, were efficient in optimizing this kind of problems. In order to verify this statement, two compound methods are proposed, where one of the two

simple methodologies generates an initial swarm-population to give it to the other one, so that the latter can finish the optimization process; see flowchart in (Figure 9). Although this increases the computer time, it guarantees a more exhaustive search through the solution space, thereby reducing the possibility of the method to lock itself up in a local optimum.

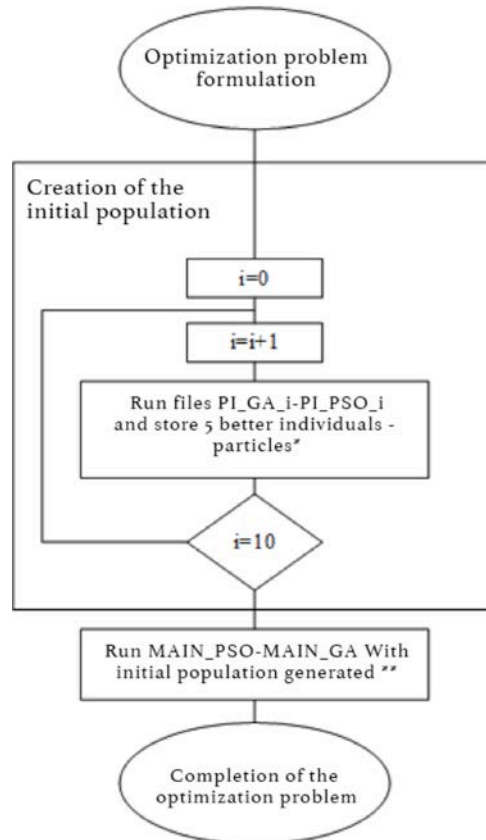


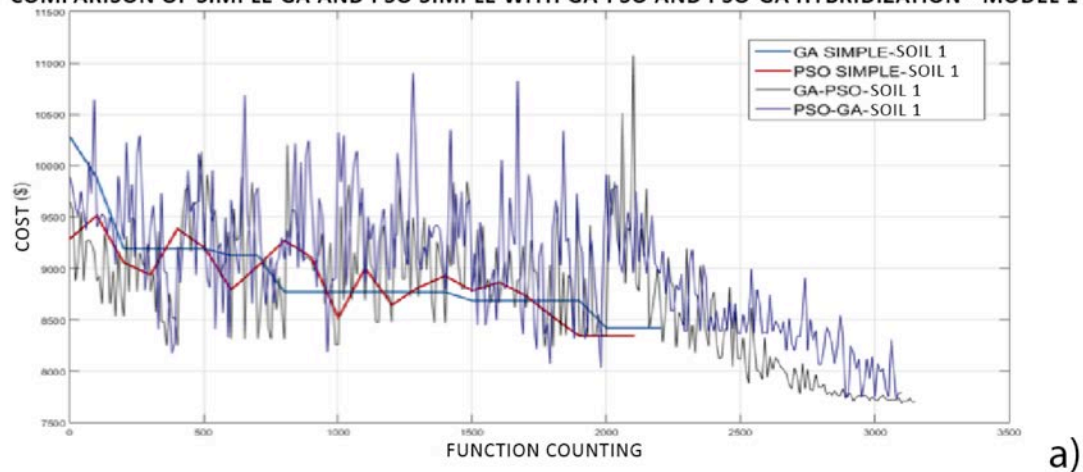
Figure 9. Flowchart of the proposed compound method, **** means that, if a simple GA is used to create the initial population, the process is optimized through simple PSO, and vice versa.

In this flowchart, PI_{GA_j} or PI_{PSO_j} represent the 10 scripts executed in the search for the best 50 points (5 per run) throughout the solution space; and both are indicated because either method can be used in this search. The optimization ends with the method that was not used for the search.

When applying both compound methods, we actually observe that simple GA and PSO methodologies are not entirely efficient in the optimization of this kind of processes; see (Figure 10). The method showing the best performance was the one using GA to create the initial population for PSO, black line in (Figure 9), which obtained economic results up

to 10% lower than the simple methods; although in this case, we do not talk about “initial population” but “initial swarm”, because the PSO does not work with “populations”, but with “swarms”. It should be highlighted that the seemingly more disorganized or confusing trend of compound methods is due to the fact that, in order to plot these processes, function count intervals are set, and the best point found is plotted; simple methods use intervals of 100 points, and compound methods use intervals of 10, which in the case of the latter reduces the possibility of plotting the result of the best values found and, obviously, this plotting procedure does not influence the final result.

COMPARISON OF SIMPLE GA AND PSO SIMPLE WITH GA-PSO AND PSO-GA HYBRIDIZATION - MODEL 1



COMPARISON OF SIMPLE GA AND PSO SIMPLE WITH GA-PSO AND PSO-GA HYBRIDIZATION - MODEL 1

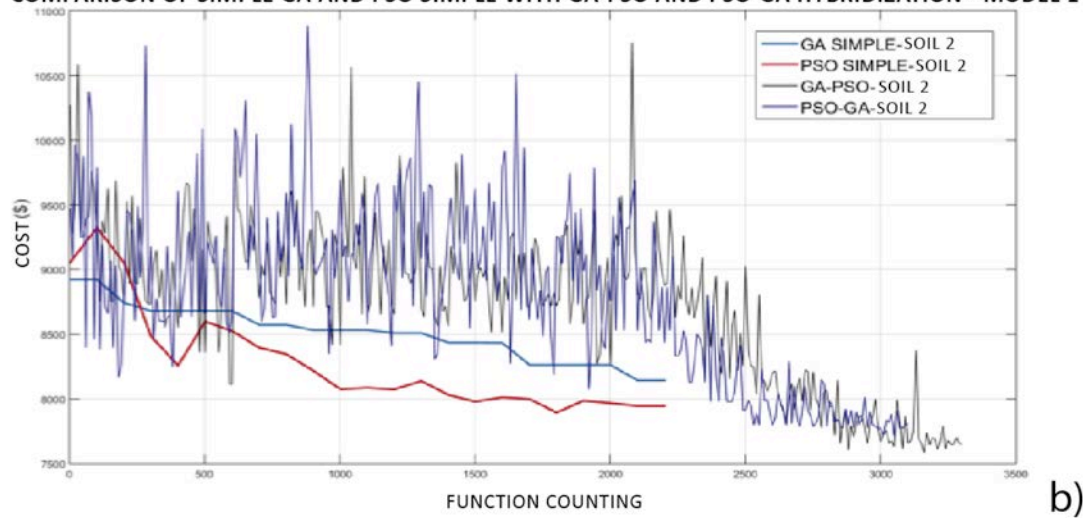


Figure 10. Comparison between simple GA and PSO with both proposed methods, a) Model 1-Soil 1, b) Model 1-Soil 2

3.1.2 Structural Results

This section presents the structural results obtained. It should be noted that only the most significant groups were selected and represented, although when the cost inputs are made, all elements are taken into account.

Regarding the beams, we will only show the results corresponding the design groups 1, 2 and 6, since they are

the most representative (Table 1). The design group 1 corresponds to the interior beams of 7-m span, group 2 to the exterior beams of the same span, and group 6 to the beams of 6-m span. This table indicates the dimensions of the cross-section of each group $h*b$ (m), as well as the distribution of the positive and negative longitudinal reinforcement (Ref. +, Ref. -).



Table 1. Comparison of results in optimized and non-optimized designs for design groups 1, 2 and 6

CASA SÍNDICO MODEL	Design groups fot optimized design		Not optimized	Optimized	
	1	h*b (m)		0.5*0.3	0.4*0.25
		Ref. +		5Ø16	5Ø16
		Ref. -		8 Ø16	9 Ø16
	2	h*b (m)		0.5*0.3	0.4*0.25
		Ref +		5Ø16	4 Ø13
		Ref -		8 Ø16	4 Ø19
	6	h*b (m)		0.5*0.3	0.4*0.25
		Ref +		4 Ø16	3 Ø16
		Ref -		4 Ø16	5Ø16

There is an interesting aspect in design groups 2 and 6, where the cross-section of the elements and the longitudinal reinforcement area (mainly positive) are simultaneously reduced, thereby evidencing that the optimization should be made to the complete structural set. Consequently, it is possible to make more rational load redistributions, that is, the variations produced in one element do not affect this one element, but the whole set.

The (Table 2) shows a comparison of results regarding the length of elements divided by the optimum height (L/h_{opt}), the optimum geometric ratio ($\rho_{geom-opt}$

$opt=As/h*b$) for the positive and negative reinforcement, the concrete compressive strength ($f'c$) for the most representative beams (6-m and 7-m span), as well as the total cost of the beams when optimizing and not optimizing the design.

When the model design is optimized, we observe that the direct cost of the beams is reduced by 13.35%. In general, we can infer that the most important element for lowering the cost is the reduction of heights (and the increase of the L/h ratio), even though this entails an increase of the geometric ratios of the longitudinal reinforcement.

Table 2. Comparison of results of L/h_{opt} , $\rho_{geom-opt}$ and $f'c$ for 6-m and 7-m span beams and the total cost of beams of optimized and non-optimized designs

CASA SÍNDICO MODEL			Not optimized	Optimized	
	L/h_{opt}	L=7m		14	17.5
		L=6m		12	15
	$\rho_{geom-opt}$ (%)	L=7m	Ref +	0.66	0.99
			Ref -	1.06	1.79
		L=6m	Ref +	0.53	0.99
			Ref -	0.53	1.55
	$f'c$ (MPa)			25	35
	Total cost of beams (\$)			13608.69	11791.34

An interesting aspect is the use of 35 MPa concrete for the beams, which contradicts the results obtained in other studies, because elements subjected to bending do not need concrete with high compressive strength. This result can be explained by the great asymmetry of the model, which increases the differential settlements of the foundations due to different load requirements to which the foundations of a same group (same dimensions) are subjected to. Therefore, the beams are the vertical stiffening element of the structure and better quality concrete guarantee a greater stiffness ($E=4700\sqrt{f'c}$).

With regard to the columns, (Table 3) summarizes the result of the non-optimized and optimized design for the three

main design groups (interior, exterior and corner), expressed in the rectangularity of the cross-section based on the span/intercolumniation ratio for the three basic groups, as well as the use of $f'c$ and the total direct cost of the construction.

The span/intercolumniation ratio is related to the length ratios of the beams that converge at the columns and, therefore, it somehow expresses in which direction there is more bending, because the longer the beam, the greater the load transfer. Moreover, it should be highlighted that only the highest ratio to which all columns are subjected to is input, because, in the end, columns are the ones offering more critical load combinations.

Table 3. Comparison of structural results of columns for optimized and non-optimized design

		Span/Intercolumniation		Not optimized	Optimized
CASA SÍNDICO MODEL	Interiors	0.73	Rectangularity (DimX/DimY)	1.00	0.7
	Outdoor	1.85	Rectangularity (DimX/DimY)	1.00	1.33
	Corner	1.85	Rectangularity (DimX/DimY)	1.00	1.33
	f'c (MPa)			25	35
	Total cost of all columns (\$)			6211.42	5332.5

The optimization of the model design shows that the main difference lies in the use of rectangular columns over square ones. Regarding the interior columns, the rectangularity goes in the “y” direction, caused by the asymmetry of the floor plan and the presence of bending in that direction. In relation to exterior and corner ones, the greatest dimension is in the direction of the greatest beam length (7 m in the “x” direction); therefore, the bending

moment for these elements goes mainly in that direction. In this case, the best performing concrete is that of 35 MPa, which enables an overall saving of 14.15% in the total direct costs of the columns.

Regarding the foundations, as in the columns, the main element to be compared is the rectangularity, as indicated in (Table 4).

Table 4. Comparison of structural results of foundations for optimized and non-optimized design, focused on their rectangularity

		Span/Intercolumniation		Not optimized	Optimized
CASA SÍNDICO MODEL	Interiors	0.73	Rectangularity (L/B)	1	0.7
	Outdoor	1.85	Rectangularity (L/B)	1	1.2
	Corner	1.85	Rectangularity (L/B)	1	1.4
	f'c (MPa)			25	20
	Total cost of all foundations (\$)			5669.35	5048.35

Something quite similar to the columns is observed here, where rectangularities are conditioned by the floor plan asymmetry, especially in interior foundations; the other groups of elements show a rectangularity in the direction of the beams with greater spans, as in the columns. In this case, the optimum concrete was that of 20 MPa, which is logical for elements with mostly flexural performance. In general, the optimization reduces the total direct cost of the foundation by 10.95%.

Normally, when applying the tool, the direct construction costs of beams, columns and foundations are reduced from \$25,489.46 to \$22,172.19, which represents a 13% saving and validates the use of the proposed

methodology. Furthermore, these structures show more efficient responses to the load requirements, since the optimization of the set allows elements to search a more rational and effective configuration to deal with the loads to which they are subjected to.

4. Conclusions

The proposed structural optimization methodology is programmed based on CSi API functions of SAP2000-MATLAB software, and it gathers several criteria usually ignored by project designers and researchers. In these processes, the PSO shows a better behavior than the GA,



although none of them offers the best results with their simple methodologies. By making a hybridization between them, using GA to create an initial swarm so that PSO can then make the optimization process, the results were up to 10% more economical. With regard to structural results, and considering the current Cuban costs, the use of beams with L/h height ratios between 15 xx and 17.5 xx is recommended, at the expense of increasing the steel geometric ratio by up to 1% for the positive reinforcement, and 1.8% for the negative one. For exterior and corner columns, the recommendation is to use rectangular sections in the direction of the highest beam span that converge to them, while for interior columns, square sections are recommended, unless the building's floor plan has a great asymmetry between continuous locals/units. And in relation to the rectangularity of the foundations, the

same as for the columns is recommended. Overall, the application of the methodology allows a 13% saving in the direct building costs, thereby validating its use.

5. Acknowledgements

Within the projects SI-VLIR "Computational Techniques for Engineering Applications" and TEAM-VLIR "Vibration Assessment of Civil Engineering Structures", postgraduate courses were taught on numerical methods, optimization, structural dynamics and other related subjects. Additionally, we had the possibility of using the software to obtain these results. We offer our sincere acknowledgement and gratitude to all the participants.

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